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# Generalized Ohm's law and potential equation in computational weakly-ionized plasmadynamics

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ABSTRACT

A variant of the generalized Ohm's law that is suited for a weakly-ionized multicomponent plasma in a magnetic field is here derived. The latter takes into consideration the current due to the non-neutrality of the plasma, the current due to the Hall effect, and the currents due to the ion slip associated with each type of ion. An equation for the electric field potential applicable to a non-neutral multicomponent plasma in the presence of a magnetic field is then presented. Despite some similarities between the potential equation and the Poisson equation, it is argued that the discretization of the potential equation cannot be accomplished in the same manner by using only central differences. It is here proven (and subsequently verified through a test case) that when the plasma exhibits conjunctly a high Hall parameter and a high electrical conductivity gradient, the centered stencils introduce spurious oscillations which can lead to severe numerical error. A novel discretization of the potential equation consisting of a blend of central and upwind differences is then presented. The proposed scheme is consistently monotonic for any value of the Hall parameter and is second-order accurate except in the vicinity of discontinuities.



## 1. Introduction

Several applications of weakly-ionized plasma technologies for improving the performance of aircraft have recently been the subject of considerable interest. One possible application is aerodynamic flow control through virtual bodies created by heat deposition using electron beams or another type of external ionizer [1,2]. Other applications are centered on the force exerted on the airflow due to magnetohydrodynamic interaction (MHD) or electrohydrodynamic interaction (EHD). The EHD interaction (or *ion wind*) is suspected to be one of the mechanisms responsible for the high success of plasma actuators in preventing or delaying boundary layer separation [3], in enhancing jet mixing [4], in keeping the flow attached on turbine blades [5], or in controlling the vortices above a delta wing [6]. On the other hand, the MHD interaction could be useful in controlling the inlet flowfield [7,8], in suppressing boundary-layer separation [9], in imparting momentum to a gas [10,11], or in generating electrical power aboard a flight vehicle through a MHD generator [12,13].

Despite some success using the weakly-ionized plasma technologies, there remain several key physical phenomena that are still not well understood. For instance, it is not clear whether plasma actuators achieve flow control through the EHD interaction or through heating, or how much of the Joule heating losses observed in a MHD generator occur within the plasma sheath. To obtain a better understanding of the physical phenomena, it is desirable to obtain more detailed computational results.

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