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Positivity-preserving flux-limited method for compressible fluid flow

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ABSTRACT

The extension of a flux discretization method to second-order accuracy can lead to some difficulties in maintaining positivity preservation. While the MUSCL-TVD scheme maintains the positivity preservation property of the underlying 1st-order flux discretization method, a flux-limited-TVD scheme does not. A modification is here proposed to the flux-limited-TVD scheme to make it positivity-preserving when used in conjunction with the Steger-Warming flux vector splitting method. The proposed algorithm is then compared to MUSCL for several test cases. Results obtained indicate that while the proposed scheme is more dissipative in the vicinity of contact discontinuities, it performs significantly better than MUSCL when solving strong shocks in hypersonic flowfields: the amount of pressure overshoot downstream of the shock is minimized and the time step can be set to a value typically two or three times higher. While only test cases solving the one-dimensional Euler equations are here presented, the proposed scheme is written in general form and can be extended to other physical models.

1. Introduction

Additionally to conservation, to monotonicity, and to the non-violation of the second-law of thermodynamics, a desirable characteristic of numerical methods for compressible flow is positivity preservation. Positivity preservation refers to the capability of an algorithm to guarantee the positivity of the internal energy and the density on all nodes as the solution advances in time, provided the initial density and internal energy are positive.

Positivity preservation of the internal energy can be especially problematic when the flow speed is in the hypersonic range with the kinetic energy composing the quasi-totality of the total energy. Indeed, since the internal energy is determined as the difference between the total and kinetic energy, a small over-estimation of the kinetic energy (or under-estimation of the total energy) by the flow solver can lead to a negative internal energy. Even with proper "clipping" of the pressure and temperature in order to keep the internal energy positive, it can then be very difficult or even impossible for the flow solver to continue the time integration process because the solution has been directed towards non-physical states.

Much attention has hence been given to determine whether the most commonly used flux discretization methods are positivity-preserving in order to assess the robustness of existing CFD codes for high speed flow. In Refs. [1,2], it is shown that while the Godunov scheme [3], the Van Leer scheme [4] and the Steger-Warming scheme [5] are positivity-preserving under a CFL-like condition, the Roe scheme [6] and the HLL schemes [7] are not. To make the Roe and HLL schemes positivity-preserving, some modifications to the original discretization stencils are proposed in Refs. [8,1].

The extension of the flux discretization method to second-order accuracy can also lead to some difficulties in maintaining positivity preservation, although some success has been reported when using Total Variation Diminishing (TVD) algorithms. Indeed, a MUSCL-TVD scheme [9] is shown in Ref. [10] to maintain the positivity preservation property of the underlying first-order method, and an attempt is made in Ref. [11] to show that a minmod-flux-limited-TVD scheme can also maintain positivity at least when the limiter is asymmetric (that is, the limiter achieves second-order accuracy for rightward traveling waves, but first-order accuracy for leftward traveling waves). However, for a symmetric minmod flux limiter achieving second-order accuracy for both leftward and rightward traveling waves, numerical tests indicate that the flux-limited-TVD scheme does not generally maintain positivity.

Because a flux-limited scheme is advantaged over MUSCL by not requiring a reconstruction of the vector of conserved variables (hence resulting in reduced computing effort), a novel modification to the flux-limiters is here presented to attain positivity preservation. A comparison is then made between the proposed algorithm and MUSCL for several test cases. To enable a fair comparison, the same underlying limiters (minmod) and the same underlying first-order monotonic method (Steger Warming Flux-Vector-Splitting) are used in both cases.

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