Journal of Computational Physics 231 (2012) 173–189

Contents lists available at SciVerse ScienceDirect

Journal of Computational Physics

journal homepage: www.elsevier.com/locate/jcp

Positivity-preserving high-resolution schemes for systems of conservation laws

Bernard Parent *

Dept. of Aerospace Engineering, Pusan National University, Busan 609-735, South Korea

A R T I C L E I N F O

Article history:
Received 5 May 2011
Received in revised form 30 August 2011
Accepted 4 September 2011
Available online 12 September 2011

Keywords:
Positivity-preservation
High resolution
Monotonicity preservation
Steger–Warming scheme
Total Variation Diminishing (TVD)
Van Leer flux limiter
Superbee flux limiter
Euler equations
Generalized curvilinear coordinates

A B S T R A C T

A new class of flux-limited schemes for systems of conservation laws is presented that is both high-resolution and positivity-preserving. The schemes are obtained by extending the Steger–Warming method to second-order accuracy through the use of component-wise TVD flux limiters while ensuring that the coefficients of the discretization equation are positive. A coefficient is considered positive if it has all-positive eigenvalues and has the same eigenvectors as those of the convective flux Jacobian evaluated at the corresponding node. For certain systems of conservation laws, such as the Euler equations for instance, this condition is sufficient to guarantee positivity-preservation. The method proposed is advantaged over previous positivity-preserving flux-limited schemes by being capable to capture with high resolution all wave types (including contact discontinuities, shocks, and expansion fans). Several test cases are considered in which the Euler equations in generalized curvilinear coordinates are solved in 1D, 2D, and 3D. The test cases confirm that the proposed schemes are positivity-preserving while not being significantly more dissipative than the conventional TVD methods. The schemes are written in general matrix form and can be used to solve other systems of conservation laws, as long as they are homogeneous of degree one.

1. Introduction

Positivity-preservation and high-resolution are two desirable attributes a flux discretization scheme should possess. High-resolution refers to the capability to capture with few nodes continuous and discontinuous waves while not introducing spurious oscillations, and can be achieved through flux or slope limiters. Positivity-preservation refers to the capability to conserve the positivity of the determinative properties. The determinative properties are the properties that must necessarily be positive for the solution to be within physical bounds. For instance, for the Euler equations, the determinative properties are the density and the temperature. For the multi-species Favre-averaged Navier–Stokes equations, not only must the density and the temperature remain positive, but the mass fractions, the turbulence kinetic energy, and the dissipation rate must also remain positive. Should the latter become negative, the solution is not within physically-admissible bounds and severe convergence difficulties can ensue. A method that is positivity-preserving prevents such convergence problems by guaranteeing positivity of the determinative properties.

When solving a scalar advection equation, the Courant upwinded stencil is well-known to conserve the positivity of the variable solved. However, when extended to second-order accuracy, the Courant scheme is not guaranteed to remain

* Tel.: +82 51 510 7338.
E-mail addresses: parent@pnu.edu, bernparent@gmail.com
URL: http://www.bernardparent.com

0021-9991/$ - see front matter
doi:10.1016/j.jcp.2011.09.006