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Sheath governing equations in computational weakly-ionized plasmadynamics

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ABSTRACT

To date, fluid models of plasma sheaths have consisted of the coupling of the electric field potential equation obtained through Gauss's law to the charged species conservation equations obtained through the drift–diffusion approximation. When discretized using finite-difference stencils, such a set of equations has been observed to be particularly stiff and to often require more than hundreds of thousands of iterations to reach convergence. A new approach at solving sheaths using a fluid model is here presented that reduces significantly the number of iterations to reach convergence while not sacrificing on the accuracy of the converged solution. The method proposed herein consists of rewriting the sheath governing equations such that the electric field is obtained from Ohm's law rather than from Gauss's law. To ensure that Gauss's law is satisfied, some source terms are added to the ion conservation equation. Several time-accurate and steady-state cases of dielectric sheaths, anode sheaths, and cathode sheaths (including glow and dark discharges) are considered. The proposed method is seen to yield the same converged solution as the conventional approach while exhibiting a reduction in computational effort varying between one-hundred-fold and ten-thousand-fold whenever the plasma includes both quasi-neutral regions and non-neutral sheaths.

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1. Introduction

In plasmadynamics, a "sheath" refers to a plasma region that is located adjacent to surface boundaries and that is significantly non-neutral. That is, a sheath always exhibits a substantial difference between its positive and negative charge densities. Several different types of sheaths are recognized, depending on the type of boundary material and also depending on the direction of the electric field near the surface. When the boundary is a dielectric, the electric field generally points towards the surface. In such a case, the plasma near the dielectric has an excess of positive charge, is a few Debye lengths thick, and is denoted as a "dielectric sheath". When the boundary is an electrode and the electric field points away from the surface, the plasma close to the electrode has an excess of negative charge and is denoted as an "anode sheath". When the boundary is an electrode and the electric field points towards the surface, the plasma close to the electrode has an excess of positive charge and is denoted as a "cathode sheath".

An accurate solution of the cathode sheath is generally crucial to predict correctly the current and electric field distribution through the rest of the plasma. This follows from the plasma near the cathode being characterized by an electron

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