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Positivity-preserving flux difference splitting schemes

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ABSTRACT

A positivity-preserving variant of the Roe flux difference splitting method is here proposed. Positivity-preservation is attained by modifying the Roe scheme such that the coefficients of the discretization equation become positive, with a coefficient considered positive if all its eigenvalues are positive and if its eigenvectors correspond to those of the flux Jacobian. Because the modification does not alter the wave speeds at the interface, the appealing attributes of the Roe flux difference splitting schemes are retained, such as high-resolution capture of discontinuous waves, low amount of artificial dissipation within viscous layers, and ease of convergence to steady-state. The proposed flux function is advantaged over previous positivity-preserving variants of the Roe method by being written in general matrix form and hence by being readily deployable to arbitrary systems of conservation laws. The stencils are extended to second-order accuracy through a newly-derived positivity-preserving total-variation-diminishing limiting process that is applied to the characteristic variables and that yields positive coefficients. Also derived is a positivity-preserving restriction on the time step for flux difference splitting schemes that is shown to depart significantly from the CFL condition in regions with high property gradients.

1. Introduction

Originally published more than three decades ago, the Roe flux difference splitting scheme [1,2] remains today one of the most used methods to discretize the convection derivatives within fluid flow systems of conservation laws. The lasting popularity of the Roe scheme lies in it having the following three properties: (i) it is monotonicity-preserving, (ii) it introduces minimal dissipation within viscous layers and discontinuities, and (iii), it is written in general matrix form. Indeed, when arithmetic averaging instead of Roe averaging is used to determine the Jacobian at the interface, the Roe flux is written in general matrix form because it is function only of the flux vector, of the vector of conserved variables, and of the eigenvalues and eigenvectors of the flux Jacobian. This makes it possible to deploy the Roe scheme, without modification, to arbitrary systems of conservation laws. Other commonly-used flux discretization approaches may have one or two of the properties just listed, but not all three. For instance, the Godunov exact Riemann solver [3], the HLLC approximate Riemann solver [4], and the AUSM method [5] are not written in general matrix form, while the Steger–Warming flux vector splitting method [6] and the HLL approximate Riemann solver [7] suffer from excessive dissipation within viscous layers.

The Roe scheme has nonetheless one major disadvantage over competing methods: it is not positivity-preserving. Positivity-preservation refers to the capability of a discretization stencil to maintain the positivity of the determinative properties, with the latter being the properties that must be positive for the solution to be physically-permissible. For instance, the determinative properties associated with the Euler equations are the density and the temperature; the determinative properties associated with the multispecies Favre-averaged Navier–Stokes equations would further include the partial densities,

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