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Positivity-preserving dual time stepping schemes for gas dynamics

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A B S T R A C T

A new approach at discretizing the temporal derivative of the Euler equations is here presented which can be used with dual time stepping. The temporal discretization stencil is derived along the lines of the Cauchy–Kowalevski procedure resulting in cross differences in spacetime but with some novel modifications which ensure the positivity of the discretization coefficients. It is then shown that the so-obtained spacetime cross differences result in changes to the wave speeds and can thus be incorporated within Roe or Steger–Warming schemes (with and without reconstruction–evolution) simply by altering the eigenvalues. The proposed approach is advantaged over alternatives in that it is positivity-preserving for the Euler equations. Further, it yields monotone solutions near discontinuities while exhibiting a truncation error in smooth regions less than the one of the second- or third-order accurate backward-difference-formula (BDF) for either small or large time steps. The high resolution and positivity preservation of the proposed discretization stencils are independent of the convergence acceleration technique which can be set to multigrid, preconditioning, Jacobian-free Newton–Krylov, block-implicit, etc. Thus, the current paper also offers the first implicit integration of the time-accurate Euler equations that is positivity-preserving in the strict sense (that is, the density and temperature are guaranteed to remain positive). This is in contrast to all previous positivity-preserving implicit methods which only guaranteed the positivity of the density, not of the temperature or pressure. Several stringent reacting and inert test cases confirm the positivity-preserving property of the proposed method as well as its higher resolution and higher computational efficiency over other second-order and third-order implicit temporal discretization strategies.

1. Introduction

First outlined in Payret and Taylor [1] and subsequently used to simulate the unsteady Euler equations in conjunction with finite-volume schemes by Jameson [2] and others [3,4], dual time stepping consists of adding a pseudotime derivative to the physical model and performing subiterations in pseudotime until the residual (consisting of the sum of the spatial and temporal derivatives) becomes small. Such a strategy is advantaged over alternate implicit integration methods by permitting the use of physical time steps that can be as high or even orders of magnitude larger than the physical time scales. Indeed, other implicit integration strategies that make use of a locally frozen Jacobian [5–8] induce considerable linearization error

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