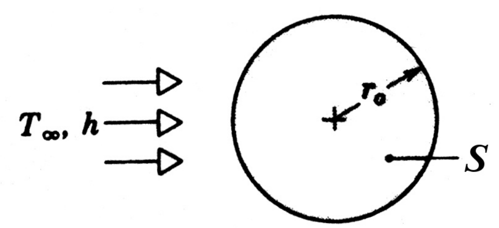
Heat Transfer Assignment 2 — 1D Steady Heat Transfer

Instructions

ξ is a parameter related to your student ID, with ξ_1 corresponding to the last digit, ξ_2 to the last two digits, ξ_3 to the last three digits, etc. For instance, if your ID is 199225962, then $\xi_1 = 2$, $\xi_2 = 62$, $\xi_3 = 962$, $\xi_4 = 5962$, etc. Keep a copy of the assignment — the assignment will not be handed back to you. You must be capable of remembering the solutions you hand in.

Question #1

Radioactive wastes are packed in a thin-walled spherical container. The wastes generate thermal energy nonuniformly according to the relation $S = S_0[1 + \xi_2(r_0/r)^2] \times [1 - (r/r_0)^2]$, where S is the local rate of energy generation per unit volume, S_0 is a constant, and r_0 is the radius of the container. Steady-state conditions are maintained by submerging the container in a liquid which is at T_∞ and provides a uniform convection coefficient h.



Obtain an expression for the total rate at which thermal energy is generated in the container. Use this result to obtain an expression for the temperature T_w of the container wall.

Question #2

A plane wall is constructed of a material having a thermal conductivity that varies as the square of temperature according to the relation $k = k_0[1 + \beta T^2]$. Derive an expression for the heat transfer in such a wall.

Question #3

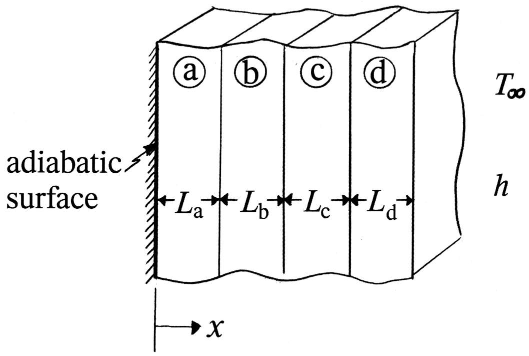
The temperature distribution in a certain plane wall is

$$\frac{T - T_1}{T_2 - T_1} = C_1 + C_2 x^2 + C_3 x^3$$

where T_1 and T_2 are the temperatures on each side of the wall. If the thermal conductivity of the wall is constant and the wall thickness is L, derive an expression for the heat generation per unit volume as a function of x, the distance from the plane where $T = T_1$. Let the heat generation be S_0 at $x = 0$.

Question #4

Consider steady-state one-dimensional heat conduction through a composite wall:



with the lengths $L_a = 0.1$ m, $L_b = 0.1$ m, $L_c = 0.1$ m, $L_d = 0.1$ m, the thermal conductivities $k_a = 20$ W/m·°C, $k_b = 50$ W/m·°C, $k_c = 10$ W/m·°C, $k_d = 5$ W/m ·°C, the heat generation per unit volume $S_a = 0$, $S_b = 10^4$ W/m^3, $S_c = 0$, $S_d = 0$, the convection heat transfer coefficient $h = 50$ W/m^2·°C, and the temperature of the environment $T_\infty = 20$° C. Do the following tasks:

(a) Sketch a qualitatively accurate temperature profile for this composite wall

(b) Find the maximum temperature in the wall.

Question #5

An electrical current of 700 Amperes flows through a stainless steel cable having a diameter of 5 mm and an electrical resistance of 6×10^{-4} Ω/m (i.e. per meter of cable length). The cable is in an environment having a temperature of 30° C, and the total coefficient associated with convection and radiation between the cable and the environment is approximately 25 W/m^2·K.

(a) If the cable is bare, what is its surface temperature?

(b) If a very thin coating of electrical insulation is applied to the cable, with a contact resistance of 0.02 m^2·K/W, what are the insulation and cable surface temperatures?

(c) There is some concern about the ability of the insulation to withstand elevated temperatures. What thickness of this insulation ($k = 0.5$ W/m · K) will yield the lowest value of the maximum insulation temperature? What is the value of the maximum temperature when this insulation thickness is used?

Answers

1. $T_\infty + \frac{2}{15} S_0 r_0 h^{-1}$.
2. $-\frac{Ak_0}{L}\left(T_2 - T_1 + \frac{\beta}{3}(T_2^3 - T_1^3)\right)$.
3. $S_0 - 3\frac{x}{L} S_0 - 6k\frac{x}{L^3}(T_2 - T_1)$.
4. 71° C.
5. 778.66° C, 778.66° C, 0.0175 m, 318.18° C.

Due on Wednesday March 27th at 9:00. Do Questions #1, #3, and #5 only.