# Heat Transfer Assignment 3 - Fins and Shapes 

## Instructions

$\xi$ is a parameter related to your student ID, with $\xi_{1}$ corresponding to the last digit, $\xi_{2}$ to the last two digits, $\xi_{3}$ to the last three digits, etc. For instance, if your ID is 199225962 , then $\xi_{1}=2, \xi_{2}=62, \xi_{3}=962, \xi_{4}=5962$, etc. Keep a copy of the assignment - the assignment will not be handed back to you. You must be capable of remembering the solutions you hand in.

## Question \#1

Fins are frequently installed on tubes by a press-fit process. Consider a circumferential aluminum fin having a thickness of 1.0 mm to be installed on a $2.5-\mathrm{cm}$-diameter aluminum tube. The fin length is 1.25 cm , and the contact conductance may be taken from the tables for a $100-\mu$ inch ground surface. The convection environment is at $20^{\circ} \mathrm{C}$, and $h=125 \mathrm{~W} / \mathrm{m}^{2} .{ }^{\circ} \mathrm{C}$. Calculate the heat transfer for each fin for a tube wall temperature of $200^{\circ} \mathrm{C}$. What percentage reduction in heat transfer is caused by the contact conductance?

## Question \#2

In certain locales, power transmission is made by means of underground cables. In one example an $8.0-\mathrm{cm}$-diameter cable is buried at a depth of 1.3 m , and the resistance of the cable is $1.1 \times 10^{-4} \Omega / \mathrm{m}$. The surface temperature of the ground is $25^{\circ} \mathrm{C}$, and $k=1.2 \mathrm{~W} / \mathrm{m} \cdot{ }^{\circ} \mathrm{C}$ for earth. Calculate the maximum allowable current if the outside temperature of the cable cannot exceed $110^{\circ} \mathrm{C}$. Hint: the heat generation in an electrical cable of length $L$ due to Joule heating is $L R_{\text {elect }} I^{2}$ in Watts with $R_{\text {elect }}$ the resistance in Ohms and $I$ the current in amperes and $L$ the length of the cable in meters.

## Question \#3

A thin rod of length $L$ and constant cross section area has its two ends connected to two walls which are maintained at temperatures $T_{1}$ and $T_{2}$, respectively. The rod loses heat to the environment at $T_{\infty}$ by convection. Derive an expression (i) for the temperature distribution in the rod and (ii) for the total heat lost by the rod through convection.

## Question \#4

Show that the fin efficiency of a fin with a rectangular cross-section and an insulated tip corresponds to:
$\eta_{\mathrm{f}}=\frac{\tanh \left(\sqrt{2} \cdot L^{1.5} \cdot\left(\frac{h}{k A_{\mathrm{m}}}\right)^{0.5}\right)}{\sqrt{2} \cdot L^{1.5} \cdot\left(\frac{h}{k A_{\mathrm{m}}}\right)^{0.5}}$
with $A_{\mathrm{m}} \equiv L \cdot t$ with $L$ the length of the fin, $t$ the thickness of the fin, $k$ the thermal conductivity, and $h$ the convective heat transfer coefficient. Outline all assumptions.

## Question \#5

Consider a micro satellite in the shape of a hollow sphere orbiting around the earth in space as follows:


Electrical circuits located within the satellite generate power with the amount $q_{\text {gen }}$ (in Watts). The temperature within either matter A or matter B can not exceed 600 K for safety reasons. The incoming radiation heat flux from the sun varies between being 0 and being $q_{\text {sun }}^{\prime \prime}=1200 \mathrm{~W} / \mathrm{m}^{2}$. The radiation heat flux from the sun may reflect on adjacent solar panels and may thus englobe the micro satellite from all directions. The thermal conductivities are of $k_{\mathrm{A}}=0.5 \mathrm{~W} / \mathrm{mK}$ and of $k_{\mathrm{B}}=0.2 \mathrm{~W} / \mathrm{mK}$, while the contact conductance between matter A and matter B is of $h_{\mathrm{c}}=24.68 \mathrm{~W} / \mathrm{m}^{2} \mathrm{~K}$. Knowing that the outer surface of the microsatellite is a black body, and that the dimensions are of $r_{1}=8 \mathrm{~cm}, r_{2}=9 \mathrm{~cm}$, $r_{3}=10 \mathrm{~cm}$, do the following:
(a) Indicate where the maximum temperature will occur (i.e. the precise location within either matter A or matter B ).
(b) Find the maximum allowable $q_{\text {gen }}$ that maintains the temperature within
both matter A and matter B to less than 600 K .
(c) Find the temperature on the outer surface of the satellite when the maximum temperature within either matter A or B is of 600 K .

## Question \#6

Consider a rectangular fin resting on a table as follows:


Knowing that the fin tip is not insulated, that $W \gg L$, and that there is no heat transfer between the fin and the table, do the following:
(a) Find the temperature at the fin $\operatorname{tip}($ at $x=L)$ as a function of $T_{0}$ and $T_{\infty}$.
(b) Find the heat transfer at the fin base (at $x=0$ )

Note: you can not assume that the thickness $t$ is much smaller than the length $L$. Outline all assumptions.

## Answers

1. 45.2 W .
2. 1181 A .
3. $q=k A m(\cosh (m L)-1)\left(T_{2}+T_{1}-2 T_{\infty}\right) / \sinh (m l)$.
4. 141.4 W .
5. $T_{\mathrm{L}}=T_{\infty}+\frac{T_{0}-T_{\infty}}{\cosh (m L)+\frac{h}{k m} \sinh (m L)}$

Due on Wednesday April 3rd at 9:00. Do Questions \#1, \#5, and \#6 only.

