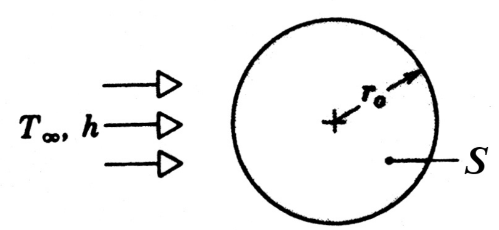
# 2010 Heat Transfer Midterm Exam (Including Solutions)

May 1st 2010
17:00 — 19:20

NO NOTES OR BOOKS; USE HEAT TRANSFER TABLES THAT WERE DISTRIBUTED; ANSWER ALL 4 QUESTIONS; ALL QUESTIONS HAVE EQUAL VALUE.

### Question #1

Radioactive wastes are packed in a thin-walled spherical container. The wastes generate thermal energy nonuniformly according to the relation $S = S_0[1 - (r/r_0)^3]$, where $S$ is the local rate of energy generation per unit volume, $S_0$ is a constant, and $r_0$ is the radius of the container. Steady-state conditions are maintained by submerging the container in a liquid which is at $T_\infty$ and provides a uniform convection coefficient $h$.




Obtain an expression for the total rate at which thermal energy is generated in the container. Use this result to obtain an expression for the temperature $T_w$ of the container wall.

### Question #2

The aircraft company you are working for is considering the use of plasma actuators to delay stall beyond the critical angle of attack. Plasma actuators can delay stall by injecting heat and applying electromagnetic forces on a region of the airflow that has been ionized. The heat injected and the applied forces alter the turbulent eddies within the boundary layer, and this can result in the flow remaining attached to the airfoil even when the angle of attack is increased beyond the critical point. In order to operate, the plasma actuators must be fed a power of 100 KiloWatts with a voltage difference of 200 Volts. You are assigned the task of designing the polyethylene-covered copper cable linking the power supply to the plasma actuators. Noting that the power supply is located 10 m away from the plasma actuators, it is desired to find the optimal cable design which minimizes weight while keeping the temperature of the polyethylene insulator below melting point. The cable is located inside the wing, where the air temperature is of $-5^\circ$ C and the convective heat transfer coefficient is known to be equal to $h = 10\ \mathrm{W/m^2 \cdot {}^\circ C}$. For safe operation the polyethylene layer is given a thickness of 0.5 cm. The electrical resistivity of copper at $20^\circ$ C can be taken as $16.8\ \mathrm{n\Omega \cdot m}$. The melting point and the thermal conductivity of polyethylene can be taken as $120^\circ$ C and $0.5\ \mathrm{W/m^\circ C}$, respectively. Design the cable with a safety margin: take into consideration that the convective heat transfer coefficient may have an error of 30% and do not let the maximum temperature within the polyethylene approach its melting point by less than $40^\circ$ C. *Hint*: assume negligible contact resistance between the copper and the polyethylene.

## Question #3

A plane wall made of a special composite material ($\rho = 2000\ \mathrm{kg/m^3}$, $c = 1000\ \mathrm{J/kg^\circ C}$, $k = 20\ \mathrm{W/m^\circ C}$) is to be used as a thermal storage unit. The thickness of this plane wall is 0.10 m, and its initial temperature is $25^\circ$ C. During the charging (energy storage) process, this wall is exposed on one of its sides to a hot gas: $T_\infty = 525^\circ\mathrm{C}$ and $h = 100\ \mathrm{W/m^2 \cdot {}^\circ C}$. The other side of the wall is very well insulated during this charging process.

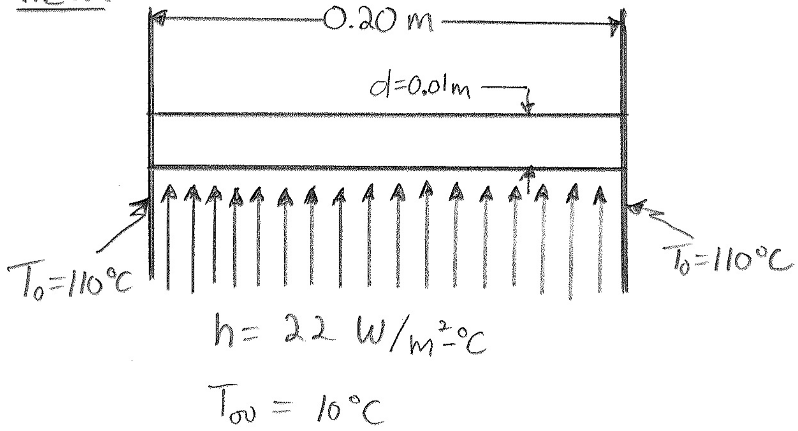
(a) How long will it take to achieve 80% of the maximum possible energy storage?

(b) At the time calculated in part (a), what is the maximum temperature in the plane wall?

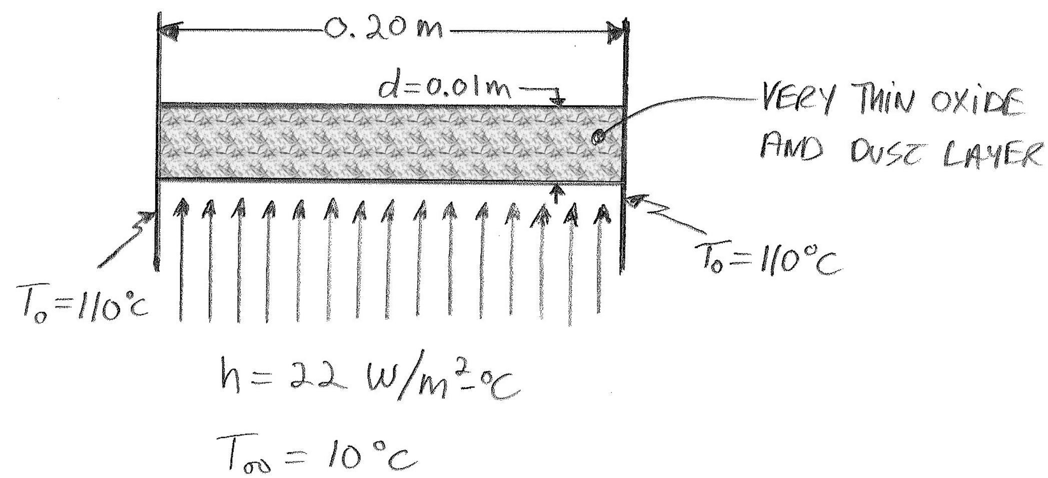
## Question #4

A slender metal rod is welded between two plates which are maintained at $110^\circ$C. At both ends of the rod, thermal contact with the plate is excellent. The rod is cooled by an air stream: $T_\infty = 10^\circ$C and $h = 22\ \mathrm{W/m^2 \cdot {}^\circ C}$. The diameter of the rod is 0.01 m, and its length is 0.20 m.

NEW:




AFTER 2 YEARS:




(a) Experimental results obtained with a very clean rod (new rod) show that the temperature at its center (midway between the walls) is 60° C. Determine the thermal conductivity of the rod metal.

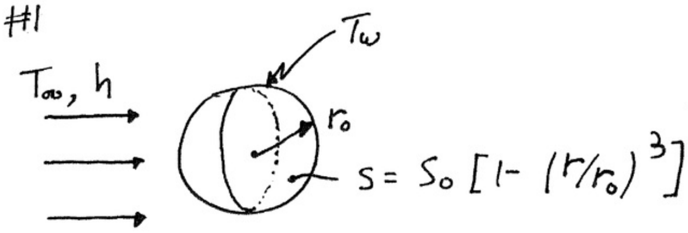
(b) After two years, a very thin oxide and dust layer develops on the *curved* (lateral) surface of the rod, and measurements indicate that the temperature of the rod at the center (midway between the walls) is 70° C. Determine the thermal contact coefficient associated with the oxide and dust layer.

## Answers

2. 0.006 m
3. 4000 s, 445 C
4. 50.74 W/m°C, 50.35 W/m°C

## Solutions

#1



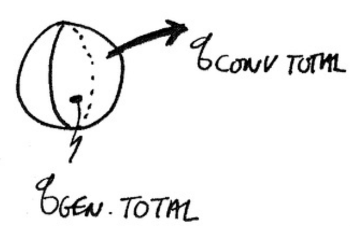

ASSUMPTIONS

- STEADY-STATE
- 1-D RADIAL PHENOMENON
- NO RADIATION
- CONSTANT h OVER SPHERE SURFACE
-

WANTED:

- EXPRESSION FOR RATE AT WHICH THERMAL ENERGY IS GENERATED INSIDE SPHERE
- OBTAIN WALL TEMPERATURE $T_w$

STEADY-STATE ENERGY BALANCE ON SPHERE:



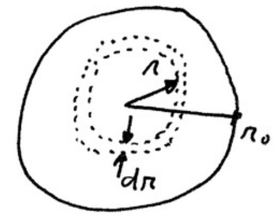

AT STEADY-STATE:

$$\boxed{q_{CONV.\ TOTAL} = q_{GEN.\ TOTAL}}$$

LET'S FIRST FIND THE RATE AT WHICH THERMAL ENERGY IS GENERATED INSIDE SPHERE ($q_{GEN.\ TOTAL}$)

AS SPECIFIED IN THE PROBLEM, THE HEAT GENERATED PER UNIT VOLUME IS ONLY A FUNCTION OF THE RADIUS. THE TOTAL HEAT GENERATED INSIDE THE SPHERE CAN HENCE BE TAKEN AS:

$$q_{\text{GEN. TOTAL}} = \int_{r=0}^{r=r_0} \underbrace{S}_{\substack{\text{HEAT GENERATED}\\ \text{PER UNIT VOLUME}}} \underbrace{4\pi r^2\, dr}_{\text{SHELL VOLUME}}$$




SINCE $S = S_0 \left[1 - (r/r_0)^3\right]$:

$$q_{\text{GEN. TOTAL}} = \int_{r=0}^{r=r_0} S_0 \left[1 - \left(\frac{r}{r_0}\right)^3\right] 4\pi r^2\, dr$$

SINCE $S_0$ IS CONSTANT, THE LATTER BECOMES:

$$q_{\text{GEN TOTAL}} = S_0\, 4\pi \int_{r=0}^{r=r_0} \left(r^2 - \frac{r^5}{r_0^3}\right) dr$$

$$= S_0\, 4\pi \left[\frac{r^3}{3} - \frac{r^6}{6 r_0^3}\right]_{r=0}^{r=r_0}$$

$$= S_0\, 4\pi \left[\frac{r_0^3}{3} - \frac{r_0^3}{6}\right]$$

$$\boxed{q_{\text{GEN TOTAL}} = S_0\, 4\pi \left[\frac{r_0^3}{6}\right]} \quad \text{(A)}$$

KNOWING THE TOTAL HEAT GENERATED INSIDE THE SPHERE, WE CAN NOW FIND THE WALL TEMPERATURE KNOWING THE CONVECTION HEAT TRANSFER COEFFICIENT ON THE SPHERE'S SURFACE, $h$, AND THE TEMPERATURE OF THE FLUID FAR FROM THE SPHERE, $T_\infty$.

INDEED, AT STEADY STATE THE TOTAL HEAT GENERATED INSIDE THE SPHERE MUST EQUAL THE HEAT LOSS THROUGH CONVECTIVE HEAT TRANSFER:

$$q_{conv.\,total} = q_{gen.\,total}$$

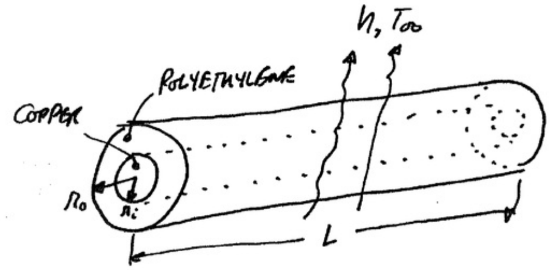
WHERE $q_{conv.\,total} \equiv A_{cs}\, h\,[T_w - T_\infty]$ AND WHERE $A_{cs} = 4\pi r_o^2$. THEN:

$$\cancel{4\pi}\, r_o^2\, h\left(T_w - T_\infty\right) = S_o\, \cancel{4\pi}\left(\frac{r_o^3}{6}\right)$$

ISOLATE $T_w$:

$$\boxed{T_w = T_\infty + \frac{S_o r_o}{6h}}$$ (A)

# #2 COPPER CABLE INSULATED BY POLYETHYLENE:




GIVEN:

- $L = 10\ m$
- $P_{ELECT.} = 100\ kW$
- $\Delta V_{ACT.} = 200\ V$
- $h = 10\ W/m^2 \, °C \pm 30\%$
- $R_{ELECT} = 16.8 \times 10^{-9}\ \Omega \cdot m$
- $T_\infty = -5°C$
- $k_{POLYETHYLENE} = 0.5\ W/m°C$
- $r_o - r_i = 0.5\ cm$
- $T_{\substack{MELTING\ POINT \\ POLYETHYLENE}} = 120°C$

WANTED:

- $r_i$ AND $r_o$ RESULTING IN MINIMUM CABLE MASS WHILE ENSURING THAT THE TEMPERATURE WITHIN THE POLYETHYLENE DOES NOT APPROACH ITS MELTING POINT BY LESS THAN 40°C.

## ASSUMPTIONS:

- STEADY-STATE
- 1D RADIAL H-T WITHIN CABLE
- $L \gg r_o$
- CONSTANT $h$ OVER CABLE SURFACE
- RADIATION H-T IS NEGLIGIBLE
- THERMAL CONDUCTIVITY OF COPPER AND POLYETHYLENE ARE CONSTANT
- NO CONTACT RESISTANCE BETWEEN COPPER AND POLYETHYLENE

## SOLUTION:

FIRST, LET'S DETERMINE THE AMOUNT OF HEAT GENERATION WITHIN THE CABLE DUE TO CURRENT FLOW.

THE CURRENT WITHIN THE CABLE CAN BE FOUND FROM:

$$I = \frac{P_{ELECT}}{\Delta V_{ACTUATOR}} = \frac{100 \times 10^3 \, W}{200 \text{ VOLTS}} = 500 \text{ Amps}$$

KNOWING THE CURRENT, WE CAN EXPRESS THE HEAT GENERATED WITHIN THE CABLE AS:

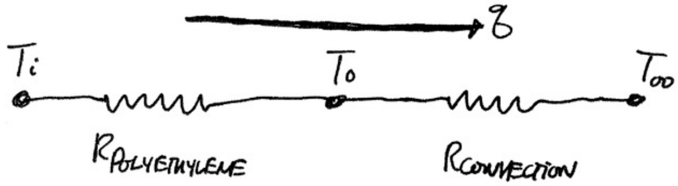
$$S_{GEN} = \frac{R_{ELECT}}{\pi r_i^2} \times I^2 \times L$$

THE LATTER CORRESPONDS TO THE HEAT GENERATED WITHIN THE ENTIRE CABLE.

THEN, AT STEADY STATE, THE HEAT TRANSFER RATE OUT OF THE CABLE MUST BE EQUAL TO $S_{GEN}$:

$$\boxed{q = S_{GEN} = \frac{R_{ELECT}}{\pi r_i^2} \times I^2 \times L} \qquad \text{I}$$

THEN, USE THE RESISTANCE ANALOGY:




USING THE CONDUCTION SHAPE FACTOR TABLE,

$$R_{POLYETHYLENE} = \frac{\ln(r_o/r_i)}{k_{POLYETHYLENE} \times 2\pi L}$$

ALSO RECALL THAT:

$$R_{CONVECTION} = \frac{1}{h\, 2\pi r_o L}$$

THEN, WE CAN RELATE $q$ TO $T_i$:

$$\boxed{q = -\frac{\Delta T}{\Sigma R} = \frac{T_i - T_\infty}{\dfrac{\ln(r_o/r_i)}{k_{POLYETHYLENE} \times 2\pi L} + \dfrac{1}{h\, 2\pi r_o L}}} \qquad \text{II}$$