2010 Heat Transfer Midterm Exam (Including Solutions)

FOR CONSTANT k, THIS YIELDS:

$$dq_{conv} = + \frac{k\pi D^2}{4} \frac{d^2T}{dx^2} dx$$

NOW USE RESISTANCE ANALOGY TO EXPRESS dq_{conv}:

$$dq_{conv} = -\frac{\Delta T}{\Sigma R} = \frac{-(T_\infty - T)}{\frac{1}{h\pi D dx} + \frac{1}{h_c \pi D dx}}$$

WITH h_c THE CONTACT CONDUCTANCE. REARRANGE:

$$dq_{conv} = \frac{(T - T_\infty)\pi D dx}{\frac{1}{h} + \frac{1}{h_c}}$$

SUBSTITUTE IN HEAT EQUATION:

$$\frac{\cancel{\pi}\cancel{D}\cancel{dx}\,(T - T_\infty)}{\frac{1}{h} + \frac{1}{h_c}} = + \frac{k\cancel{\pi}D^{\cancel{2}}}{4} \frac{d^2T}{dx^2} \cancel{dx}$$

OR

$$T - T_\infty = + \left(\frac{1}{h} + \frac{1}{h_c}\right) \frac{kD}{4} \frac{d^2T}{dx^2}$$

NOW DEFINE

$$\theta \equiv T - T_\infty$$

$$m^2 \equiv \frac{4}{kD\left(\frac{1}{h} + \frac{1}{h_c}\right)}$$

THEN THE HEAT EQ. BECOMES:

$$\frac{d^2\theta}{dx^2} = \theta m^2$$

FIND SOLUTION USING H-T TABLES:

$$\boxed{\theta = A \sinh(mx) + B \cosh(mx)}$$

APPLY B.C.s TO FIND A AND B; AT $x=0$, $T = T_w$; THIS YIELDS:

$$\boxed{T_w - T_\infty = B}$$

ALSO, DUE TO SYMMETRY, AT $x = L/2$, $d\theta/dx = 0$:

$$\left.\frac{d\theta}{dx}\right|_{x=L/2} = 0 = Am \cosh\left(\frac{mL}{2}\right) + Bm \sinh\left(\frac{mL}{2}\right)$$

ISOLATE A:

$$\boxed{A = -B \tanh\left(\frac{mL}{2}\right)}$$

SUBSTITUTE A AND B IN GENERAL SOLUTION:

$$\theta = -B \tanh\left(\frac{mL}{2}\right)\sinh(mx) + B\cosh(mx)$$

SUBSTITUTE $\theta = T - T_\infty$ AND $B = T_w - T_\infty$:

$$T - T_\infty = -(T_w - T_\infty)\tanh\left(\frac{mL}{2}\right)\sinh(mx) + (T_w - T_\infty)\cosh(mx)$$

NOW OBTAIN SOLUTION FOR T_m AT $x = L/2$:

$$\frac{T_m - T_\infty}{T_w - T_\infty} = \cosh\left(\frac{mL}{2}\right) - \tanh\left(\frac{mL}{2}\right)\sinh\left(\frac{mL}{2}\right)$$

MULTIPLY BY $\cosh\left(\frac{mL}{2}\right)$ BOTH SIDES:

$$\left(\frac{T_m - T_\infty}{T_w - T_\infty}\right)\cosh\left(\frac{mL}{2}\right) = \cosh^2\left(\frac{mL}{2}\right) - \sinh^2\left(\frac{mL}{2}\right)$$

BUT $\sinh^2(x) - \cosh^2(x) = -1$:

$$\frac{T_m - T_\infty}{T_w - T_\infty} = \frac{1}{\cosh\left(\frac{mL}{2}\right)}$$

OR

$$\frac{mL}{2} = \cosh^{-1}\left(\frac{T_w - T_\infty}{T_m - T_\infty}\right)$$

BUT RECALL THAT

$$m = \sqrt{\frac{4}{kD\left(\frac{1}{h}+\frac{1}{h_c}\right)}}$$

SUBSTITUTE IN FORMER:

$$\frac{\cancel{4}}{kD\left(\frac{1}{h}+\frac{1}{h_c}\right)} = \frac{\cancel{4}}{L^2}\left[\cosh^{-1}\left(\frac{T_w-T_\infty}{T_m-T_\infty}\right)\right]^2$$

OR:

$$\boxed{k\left(\frac{1}{h}+\frac{1}{h_c}\right) = \frac{L^2}{D}\left[\cosh^{-1}\left(\frac{T_w-T_\infty}{T_m-T_\infty}\right)\right]^{-2}} \quad I$$

ISOLATE k IN EQ. I:

$$k = \frac{L^2}{D\left(\frac{1}{h}+\frac{1}{h_c}\right)}\left[\cosh^{-1}\left(\frac{T_w-T_\infty}{T_m-T_\infty}\right)\right]^{-2}$$

AND FOR NO CONTACT CONDUCTANCE ; $T_w = 110°C$, $T_m = 60°C$:

$$k = \frac{(0.20\,m)^2}{0.01\,m} \times \frac{22\,W}{m^2\,°C} \times \left[\cosh^{-1}\left(\frac{110°C-10°C}{60°C-10°C}\right)\right]^{-2}$$

$$\boxed{k = 50.74\ \frac{W}{m°C}}$$ (A)

NOW, KNOWING k, ISOLATE h_c IN EQ. I:

$$\frac{1}{h} + \frac{1}{h_c} = \frac{L^2}{kD}\left[\cosh^{-1}\left(\frac{T_w - T_\infty}{T_m - T_\infty}\right)\right]^{-2}$$

$$h_c = \frac{1}{-\frac{1}{h} + \frac{L^2}{kD}\left[\cosh^{-1}\left(\frac{T_w - T_\infty}{T_m - T_\infty}\right)\right]^{-2}}$$

SUBSTITUTE VALUES: $T_w = 110°C$, $T_m = 70°C$, $T_\infty = 10°C$,
$k = 50.74\ W/m°C$, $D = 0.01m$,
$h = 22\ W/m^2°C$

$$h_c = \frac{1}{\frac{-1}{22\ W/m^2°C} + \frac{(0.20m)^2}{50.74\ W/m°C \times 0.01\ m} \times \left[\cosh^{-1}\left(\frac{110°C - 10°C}{70°C - 10°C}\right)\right]}$$

THIS YIELDS:

$$\boxed{h_c = 50.35\ W/m^2°C}$$ (A)

HOW THE MIDTERM IS SCORED:

QUESTION 1:

- 5 PTS ASSUMPTIONS
- 10 PTS TO FIND $q_{GEN\ TOTAL} = S_0 4\pi (r_0^3/6)$
- 5 PTS TO FIND $q_{CONV.\ TOTAL} = q_{GEN\ TOTAL}$ AT S-S
- 5 PTS TO FIND $T_W = T_\infty + S_0 r_0 / 6h$

QUESTION 2:

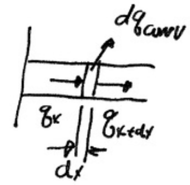
- 6 PTS ASSUMPTIONS
- 5 PTS TO FIND $q = \dfrac{T_i - T_\infty}{\dfrac{\ln(r_0/r_i)}{k_{POLY}\, 2\pi L} + \dfrac{1}{h\, 2\pi r_0 L}}$
- 5 PTS TO FIND $q = S_{GEN} = \dfrac{R_{ELECT}\, I^2 L}{\pi r_i^2}$
- 4 PTS TO FIND $h = 7\ W/m^2\,°C$ AND $T_i = 80°C$
- 5 PTS TO OBTAIN $r_i = 6\ mm$

QUESTION 3

- 5 POINTS FOR ASSUMPTIONS
- 5 POINTS TO FIND $Q/Q_o = 0.8$
- 3 POINTS TO FIND $Bi^2 Fo = 1.0$ FROM HEISLER CHARTS KNOWING $Bi = 0.5$ AND $Q/Q_o = 0.8$
- 3 POINTS TO FIND $t = 4000\ s$ FROM $Bi^2 Fo = 1.0$
- 3 POINTS TO FIND $Fo = 4.0$ (AND $x = L$ FOR $T = T_{MAX}$)
- 4 POINTS TO STATE $\frac{T_{MAX} - T_\infty}{T_i - T_\infty} = \underbrace{\frac{T_o - T_\infty}{T_i - T_\infty}}_{\text{MIDPLANE TEMP. CHART}} \times \underbrace{\frac{T_{MAX} - T_\infty}{T_o - T_\infty}}_{\text{TEMP. DISTRIBUTION CHART.}}$
- 2 POINTS TO FIND $\frac{T_{MAX} - T_\infty}{T_i - T_\infty} = 0.2 \times 0.8$ AND $T_{MAX} = 445°C$

QUESTION 4

- 5 POINTS FOR ASSUMPTIONS
- 4 POINTS TO OUTLINE:



$$q_x = q_{x+dx} + dq_{conv.}$$

- 2 POINTS TO STATE
  - $q_x = -k \frac{dT}{dx} \frac{\pi D^2}{4}$
  - $q_{x+dx} = -k\frac{dT}{dx}\frac{\pi D^2}{4} - \frac{d}{dx}\left(\frac{k\pi D^2}{4}\frac{dT}{dx}\right)dx + \ldots$
- 2 POINTS TO STATE:

$$dq_{conv} = \frac{-\Delta T}{\Sigma R} = \frac{-(T_\infty - T)}{\frac{1}{h\pi D dx} + \frac{1}{h_c \pi D dx}}$$

- 2 POINTS TO FIND
  - $\theta = A\sinh(mx) + B\cosh(mx)$
  - $m^2 = \frac{4}{kD\left(\frac{1}{h} + \frac{1}{h_c}\right)}$
- 2 POINTS FOR B.C. @ $x=0$, $T=T_w$
- 2 POINTS FOR B.C. @ $x = L/2$, $d\theta/dx = 0$
- 2 POINTS TO OBTAIN $\frac{T_m - T_\infty}{T_w - T_\infty} = \frac{1}{\cosh(mL/2)}$
- 2 POINTS TO FIND $k = 50.74$ W/m°C
- 2 POINTS TO FIND $h_c = 50.35$ W/m²°C