Heat Transfer Handout 2 — Natural Convection

THEN:

$$\frac{\delta}{x} \simeq 3.93 \left(\frac{20}{21} + Pr\right)^{1/4} \left(\frac{g\,\beta\,(T_w - T_\infty)\,x^3}{\nu^2}\right)^{-1/4} \left(Pr\right)^{-1/2}$$

DEFINE GRASHOF NUMBER:

$$Gr_x \equiv \frac{g\,\beta\,(T_w - T_\infty)\,x^3}{\nu^2}$$

THEN

$$\boxed{\frac{\delta}{x} \simeq 3.93\, Pr^{-1/2} (0.952 + Pr)^{1/4}\, Gr_x^{-1/4}}$$

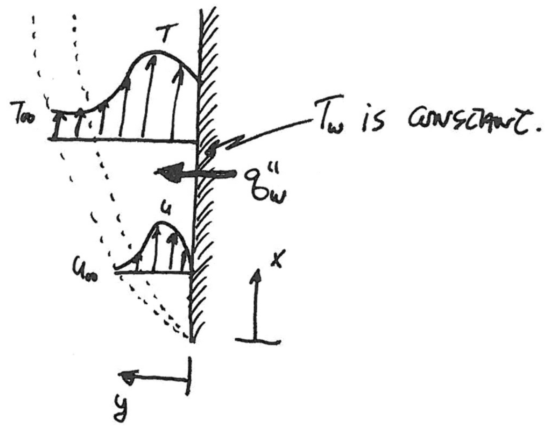
ASSUMPTIONS:

- $\delta = \delta_t$ } SIGNIFICANT ERROR FOR $Pr \gg 1$
- CONSTANT T_{WALL}
- NO VISCOUS DISSIPATION
- $u_x = C_1 x^{1/2} + \cancel{\ldots}$ neglect } SIGNIFICANT ERROR
- $\delta = C_2 x^{1/4} + \cancel{\ldots}$ neglect } ??
- T is A 2nd DEGREE POLYNOMIAL } POSSIBLE LARGE ERROR
- u is A 3rd DEGREE POLYNOMIAL
- $\left|\frac{T - T_\infty}{T_\infty}\right| \ll 1$

FREE CONVECTION HEAT TRANSFER (VERTICAL PLATE)

(GO QUICKLY – GIVE HANDOUT)

RECALL THE THERMAL AND BOUNDARY LAYER FORMED BY FREE CONVECTION OVER A VERTICAL FLAT PLATE:



WE SHOWED THAT THE TEMPERATURE PROFILE CORRESPONDS APPROXIMATELY TO:

$$\frac{T-T_\infty}{T_w-T_\infty} = \left(1-\frac{y}{\delta}\right)^2 \qquad \text{I}$$

AND WE SHOWED THAT

$$\frac{\delta}{x} = 3.93\, Pr^{-1/2}\left(0.952+Pr\right)^{1/4} Gr_x^{-1/4} \qquad \text{II}$$

NOW, RECALL THAT THE HEAT FLUX AT THE WALL CORRESPONDS TO:

$$q_w'' = -k \left.\frac{\partial T}{\partial y}\right|_{y=0}$$

THEN FIND $\partial T/\partial y$ BY TAKING THE DERIVATIVE WITH RESPECT TO y ON BOTH SIDES OF EQ. [redacted] I:

$$\frac{\partial T}{\partial y} = 2(T_w - T_\infty)\left(1 - \frac{y}{\delta_{\text{[redacted]}}}\right)\left(\frac{-1}{\delta_{\text{[redacted]}}}\right)$$

AT $y=0$, THE LATTER YIELDS:

$$\left.\frac{\partial T}{\partial y}\right|_{y=0} = -\frac{2(T_w - T_\infty)}{\delta_{\text{[redacted]}}}$$

SUBSTITUTE THE LATTER IN THE EQ. FOR q_w'':

$$q_w'' = \frac{2k}{\delta_{\text{[redacted]}}}(T_w - T_\infty)$$

THEN RECALL THAT THE DEFINITION OF THE CONVECTIVE HEAT TRANSFER COEFFICIENT YIELDS:

$$q_w'' = h(T_w - T_\infty)$$

COMPARING THE LATTER WITH THE FORMER IT IS EVIDENT THAT:

$$h = \frac{2k}{\delta}$$

THEN RECALL THE DEFINITION OF THE NUSSELT NUMBER:

$$Nu_x \equiv \frac{hx}{k}$$

SUBSTITUTE THE FORMER IN THE LATTER:

$$Nu_x = \frac{2\cancel{k}}{\cancel{k}} \frac{x}{\delta}$$

THEN SUBSTITUTE δ/x FROM EQ. II:

$$Nu_x = \frac{2}{3.93} Pr^{1/2} (0.952 + Pr)^{-1/4} Gr_x^{1/4}$$

OR

$$\boxed{Nu_x \approx 0.5\, Pr^{1/2} (0.952 + Pr)^{-1/4} Gr_x^{1/4}} \qquad \text{III}$$

NOW NOTE THE FOLLOWING:

IF $Pr \sim 1$ THEN $(0.952 + Pr)^{-1/4} \sim 1 \sim Pr^{-1/4}$

IF $Pr \gg 1$ THEN $(0.952 + Pr)^{-1/4} \sim Pr^{-1/4}$

THEREFORE WE CAN SAY:

$$(0.952 + Pr)^{-1/4} = Pr^{-1/4} \qquad \text{IV}$$

IF $Pr \gtrsim 1$. FOR $Pr \sim 0.7$ THE LATTER ENTAILS AN ERROR OF 12% OR SO. SUCH AN ERROR IS SMALL COMPARED TO THE ERROR ORIGINATING FROM THE ASSUMPTIONS MADE IN DERIVING EQ. III. THEN SUBSTITUTE IV IN III:

$$Nu_x \approx 0.5\, Pr^{1/2}\, Pr^{-1/4}\, Gr_x^{1/4}$$

THIS YIELDS

$$\boxed{Nu_x \approx 0.5\, Pr^{1/4}\, Gr_x^{1/4}}$$

NOW DEFINE THE RAYLEIGH NUMBER AS:

$$Ra_x \equiv Gr_x Pr$$

THEN THE NUSSELT # BECOMES

$$\boxed{Nu_x = 0.5\, Ra_x^{1/4}}$$

FURTHER, IT CAN BE EASILY SHOWN THAT THE AVERAGE NUSSELT # OVER THE LENGTH OF THE PLATE CORRESPONDS TO 4/3 THE LOCAL NUSSELT # AT $x = L$:

$$\overline{Nu_L} = \frac{4}{3} \times 0.5 \times Ra_L^{1/4}$$

OR

$$\boxed{\overline{Nu_L} = 0.67\, Ra_L^{1/4}}$$

PHYSICAL SIGNIFICANCE OF RAYLEIGH #:

$$Ra = \frac{\text{BUOYANCY FORCES}}{\text{THERMAL} \times \text{MOMENTUM DIFFUSIVITY}}$$

$$Gr = \frac{\text{BUOYANCY FORCE}}{\text{VISCOUS FORCE}}$$

FREE CONVECTION H-T OVER COMPLEX GEOMETRIES

AS WAS SHOWN IN THE PREVIOUS SECTION, H-T OVER A VERTICAL FLAT PLATE DUE TO FREE CONVECTION CAN BE FOUND FROM

$$Nu_x = 0.5\, Ra_x^{1/4}$$

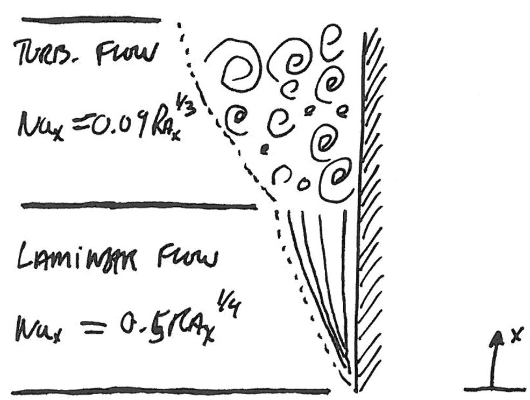
AND

$$\overline{Nu_L} = 0.67\, Ra_L^{1/4}$$

BUT THE LATTER IS ONLY APPLICABLE FOR LAMINAR FLOW OVER A FLAT PLATE.

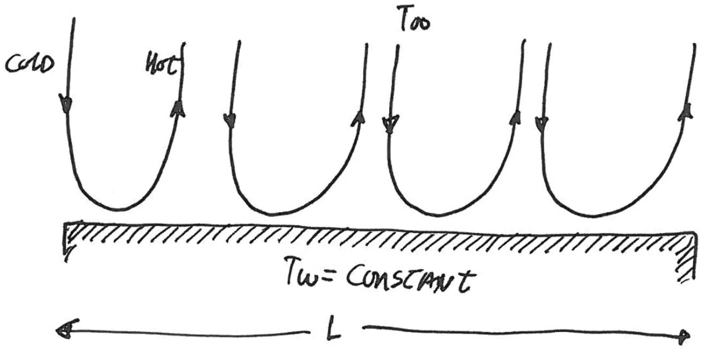
FOR TURBULENT FLOW, EXPERIMENTS INDICATE THAT:

$$Nu_x \simeq 0.09\, Ra_x^{1/3}$$



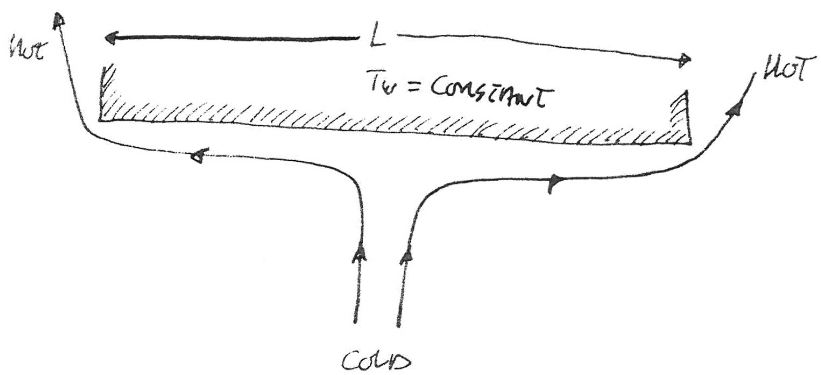
BUT, WHAT ABOUT OTHER GEOMETRIES?

UPPER SURFACE OF HEATED PLATE: ($T_w > T_\infty$)



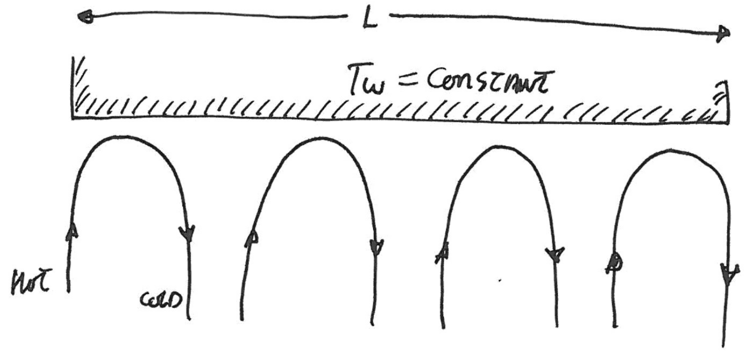
$$Nu_L = 0.15\,(Ra_L)^{1/3} \quad \text{FOR} \quad 8 \times 10^6 < Ra_L < 10^{11}$$

LOWER SURFACE OF HEATED PLATE: ($T_w > T_\infty$)



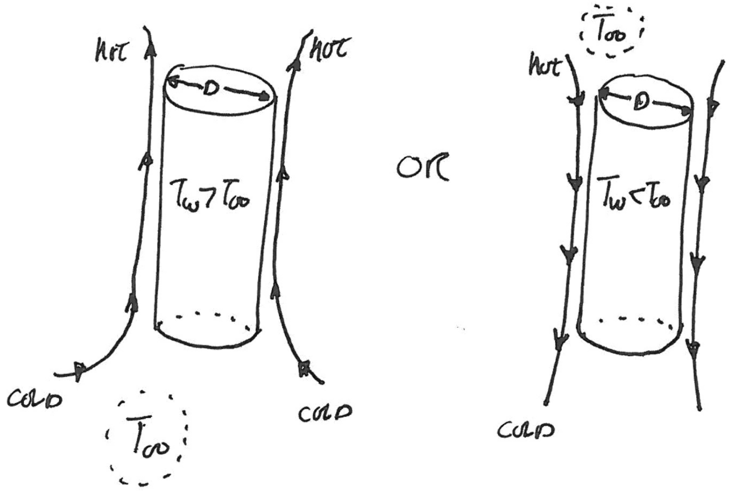
$$Nu_L = 0.27\,(Ra_L)^{1/4} \quad \text{FOR} \quad 10^5 < Ra_L < 10^{11}$$

LOWER SURFACE OF COOLED PLATES: ($T_w < T_\infty$)



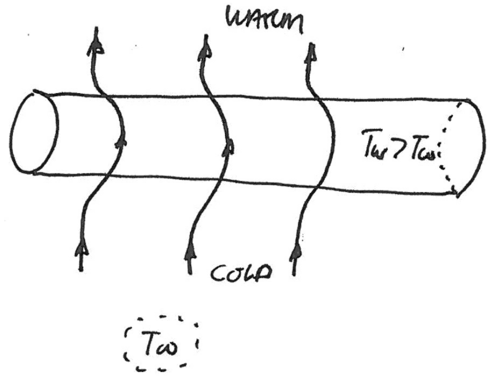
$$Nu_L = 0.15\,(Ra_L)^{1/3} \quad \text{FOR} \quad 8 \times 10^6 < Ra_L < 10^{11}$$

VERTICAL CYLINDER ($T_w < T_\infty$ OR $T_w > T_\infty$):

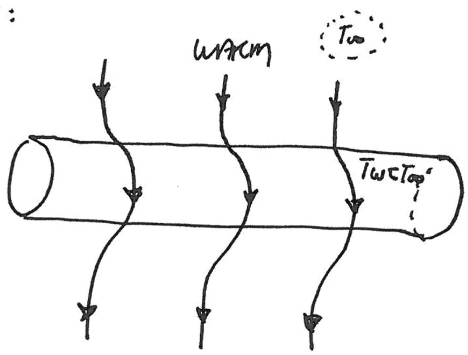


$$Nu_D = 0.775\,(Ra_D)^{0.21} \quad \text{FOR} \quad 10^4 < Ra_D < 10^8$$

HORIZONTAL CYLINDERS ($T_w < T_\infty$ OR $T_w > T_\infty$)



OR:



THEN USE:

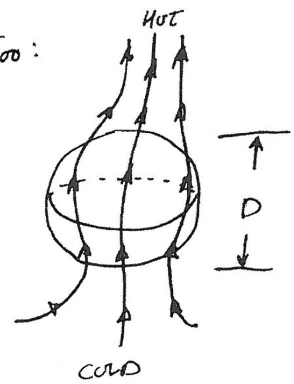
$$Nu_D^{0.5} = 0.6 + 0.387 \left[\frac{Ra_D}{\left[1 + (0.559/Pr)^{9/16} \right]^{16/9}} \right]^{1/6}$$

WIDE RANGE OF APPLICABILITY:

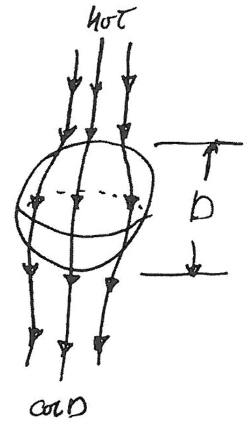
$$10^{-5} < Ra_D < 10^{12}$$

SPHERE ($T_w < T_\infty$ OR $T_w > T_\infty$)

$T_w > T_\infty$:



$T_w < T_\infty$:

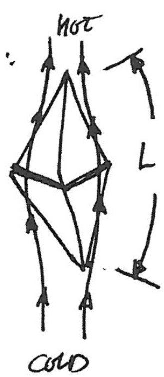


$$Nu_D = 2 + 0.43\,(Ra_D)^{1/4} \quad \text{FOR} \quad 1 < Gr_D < 10^5$$

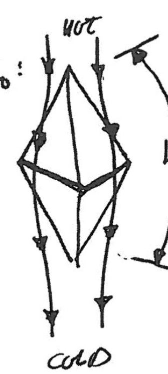
$$Nu_D = 2 + 0.5\,(Ra_D)^{1/4} \quad \text{FOR} \quad 3\times10^5 < Ra_D < 8\times10^8$$

IRREGULAR SOLIDS ($T_w < T_\infty$ OR $T_w > T_\infty$)

$T_w > T_\infty$:



$T_w < T_\infty$:



$$Nu_L = 0.52\,(Ra_L)^{1/4}$$

$L \equiv$ CHARACTERISTIC DISTANCE A FLUID PARTICLE TRAVELS IN BOUNDARY LAYER