CFDWARP — Positivity-Preserving FDS

THE CONVECTION FLUXES ARE DISCRETIZED in CFDWARP using the Roe solver modified to be positivity-preserving through a novel approach developed by our group in [1,2,3].

Positivity preservation refers to the capability of a numerical method to conserve the positivity of the internal energy and the density. If the method cannot conserve the positivity of the energy and density, there is a chance that the solution may be directed towards aphysical states, which can lead to some difficulties in reaching convergence. Along with monotonicity-preservation and flux conservation, positivity preservation is an important feature of a numerical method especially at high Mach number where negative internal energies and densities are more likely to occur.

However, positivity preservation is not trivial to obtain and the flux discretization algorithms that are the most commonly used in compressible flow codes are not positivity preserving. For instance, when solving the Euler equations, the Roe approximate Riemann solver (one of the most popular schemes for compressible flow) is not positivity preserving, and any method using TVD flux limiters to reach second-order accuracy is also not positivity preserving.

Recently, a new property of the system of compressible flow equations was discovered by our laboratory that can help craft positivity-preserving stencils. As first shown by our group in [1], a discretization stencil solving the Euler equations is positivity preserving if all coefficients within the discretization equation are positive, with a coefficient considered positive if all its eigenvalues are positive and if its eigenvectors are the same as those of the respective flux Jacobian. Named the "rule of the positive coefficients", such was used to craft a new positivity-preserving variant of the TVD schemes using the Roe solver by our lab in Refs. [2] and [3].

The in-house-developed positivity-preserving Roe solver that is used in our code is advantaged over previous attempts at making the Roe scheme positivity-preserving by being written in general matrix form and hence being readily deployable to arbitrary systems of conservation laws, such as those encountered in non-equilibrium gasdynamics and plasmadynamics.

References

[1] B Parent. "Positivity-preserving Flux-Limited Method for Compressible Fluid Flow", *Computers and Fluids*, 44 (1), Pages 238-247, 2011. [pdf]

[2] B Parent. "Positivity-preserving High-resolution Schemes for Systems of Conservation

Laws", *Journal of Computational Physics*, 231 (1), Pages 173-189, 2012. [pdf]

[3] B Parent. "Positivity-preserving Flux Difference Splitting Schemes", *Journal of Computational Physics*, 243, Pages 194-209, 2013. [pdf]