Compressible Flow Design Problem Set
1 — Supersonic Wind Tunnels and Diffusers

Instructions

ξ is a parameter related to your student ID, with ξ1 corresponding to the last digit, ξ2 to the last two digits, ξ3 to the last three digits, etc. For instance, if your ID is 199225962, then ξ1 = 2, ξ2 = 62, ξ3 = 962, ξ4 = 5962, etc. Keep a copy of the assignment — the assignment will not be handed back to you. You must be capable of remembering the solutions you hand in.

Design Problem #1

Design a continuous-flow supersonic wind tunnel to simulate the conditions $M = 2.5$, $P = 30$ kPa and $T = 230$ K with the best possible performance. The test section must have a cross-section area of 180 cm$^2$ and is followed by a fixed-geometry diffuser. Specifically, find the diffuser and nozzle throat areas and the mass flux through the system necessary to start and operate the system. At operating conditions, sketch the Mach number distribution along the wind tunnel from the nozzle inlet to just past the diffuser throat.

Design Problem #2

A fixed-geometry, convergent-divergent wind tunnel diffuser is to be designed for Mach number 2. Assuming no friction, compare the minimum possible percent loss in stagnation pressure during operation for the following cases:
(a) The best possible design is employed
(b) The design is conservative, with a throat area 5 percent larger than that required for starting, and with the shock located during operation at an area 5 percent greater than the throat area.
(c) The converging portion is eliminated, and the process comprises a normal shock followed by reversible subsonic compression.

Design Problem #3

A ramjet is to fly at 12,000 m altitude with a speed of 3200 km/hr.
(a) Design the best fixed-geometry, convergent-divergent diffuser for this aircraft, and compute for it the least percent loss in stagnation pressure.

(b) Suppose that it were possible to overspeed the aircraft to 3840 km/hr. Design the best convergent-divergent diffuser which could then be used, and find for it the least percent loss in stagnation pressure at the operating speed of 3200 km/hr.

Design Problem #4

Consider that you are faced with designing a nozzle for a supersonic wind tunnel which is to have an operating test section Mach number of $M = 2$ and a circular test cross-section of approximately 10 cm in diameter.
(a) As a zero order iteration, you decide to start out by machining a simple converging-diverging conical nozzle to get some feel for what's required:

The nozzle is equipped with a static pressure hole in the exit plane. The air for the system is supplied from a large reservoir at a pressure of 10 bar and room temperature of 20° C (293 K). The supersonic nozzle (and eventually the wind tunnel) will exhaust into a large partially evacuated tank at a pressure in the range of approximately 0.1 bar. For the above conical nozzle under the specified operating conditions, pitot tube-static pressure measurements in the test section near the exit plane indicate pressures of $P_{pitot \ tube} = 5.6677$ bar $P_{static \ hole} = 1.1023$ bar

Determine the exit Mach number and the nozzle efficiency.

(b) Based on these preliminary tests, you are now in a position to machine out the conical nozzle to a final contoured converging-diverging supersonic de Laval nozzle, as shown here:
For the same intake reservoir and exhaust tank conditions, you now find that the pitot-static pressure measurements in the exit plane give a pressure ratio of:

\[ P_{\text{pitot tube}} = 7.2084 \text{ bar} \]
\[ P_{\text{static hole}} = 1.2780 \text{ bar} \]

Determine the exit Mach number and the nozzle efficiency.

(c) The final contoured de Laval nozzle, described in (b), is now put through its paces by testing it over a whole range of intake and exhaust conditions. In one such test, the reservoir pressure is 15 bar and the temperature is 20°C. A pitot tube measurement in the exit plane gives a pressure of 11.65 bar. Determine the exit Mach number, the exit pressure and sketch out roughly the Mach number distribution along the length of the nozzle from the throat to the exit.

(d) The nozzle with its test section is now connected to a supersonic diffuser and exhauster into the large dump tank to form an operational supersonic wind tunnel system. The operational intake reservoir conditions are now again at a pressure of 15 bar and a temperature of 20°C in the reservoir. Determine the necessary geometric conditions for starting-up the wind tunnel and the diffuser exit conditions for the optimal operation of the wind tunnel (e.g. maximum pressure recovery), if the diffuser exit Mach number \( M_D = 0.2 \).

Design Problem #5

A supersonic aircraft is equipped with a two-dimensional, converging-diverging, variable throat area, intake diffuser. The diffuser is designed for a cruise Mach number of 2.2. What percent increase in throat area is required to “swallow” the shock? If in the takeoff sequence the aircraft has to “boiler” at Mach 1.8 due to tactical reasons, what percent of mass spill of air occurs, with the rest, of course, passing through the engine, if the diffuser configuration happens to be set for cruise at \( M_\infty = 2.2 \)?

Design Problem #6

Consider a continuous supersonic wind tunnel as illustrated below:

The test section has a cross-sectional area of 5 m², and the wind tunnel should be designed such that the pressure in the test section is 0.1 atm and the Mach number in the test section is 2.5. Perform the following tasks:

(a) For a fixed-geometry nozzle and a fixed-geometry diffuser, find the nozzle throat area \( A_t \) and the diffuser throat area \( A_d \), and sketch the Mach number distribution between stations 1 and 6.

(b) For a fixed-geometry nozzle and a variable-geometry diffuser, find the nozzle throat area and the minimum and maximum diffuser throat area. As well, sketch the Mach number and pressure distribution between stations 1 and 6 when the wind tunnel operates at maximum efficiency.

(c) What role does the compressor play for a fixed-geometry diffuser? What role does the compressor play for a variable geometry diffuser?

Design Problem #7

Consider a supersonic wind tunnel. When the tunnel is operating in normal conditions, a shock sits at the throat of the diffuser as follows:

Knowing that the pressure ratio across this shock is of 1.8, and knowing that the throat area of the diffuser needs to be increased by a factor of 1.5 when starting the tunnel (i.e. \( A_t_{\text{startup}} = 1.5 A_t_{\text{normal}} \)), find the Mach number in the test section under normal operating conditions.

Answers

1. 6.2 kg/s, 68 cm², 137 cm².
2. 27.9%. 
4. (a) $M = 1.90$, $\eta_{\text{ideal}} = 0.94$; (b) $2.0$; (c) $0.52$, 9.7 bar; (d) $66.6 \text{ cm}^2$, 1.75, 12.2 bar, 291 K.

5. 59.2%, 41.7%.

7. 2.21.

Due on Tuesday October 30th at 16:30. Do Problems #2 and #5 only.