Compressible Flow Design Problem Set 1 — Supersonic Wind Tunnels and Diffusers

Instructions

ξ is a parameter related to your student ID, with ξ1 corresponding to the last digit, ξ2 to the last two digits, etc. For instance, if your ID is 199025692, then ξ1 = 2, ξ2 = 62, ξ3 = 926, ξ4 = 562, etc. Keep a copy of the assignment — the assignment will not be handed back to you. You must be capable of remembering the solutions you hand in.

Design Problem #1

Design a continuous-flow supersonic wind tunnel to simulate the conditions $M = 2.5$, $P = 30$ kPa and $T = 330$ K with the best possible performance. The test section must have a cross-sectional area of 380 cm² and is followed by a fixed-geometry diffuser. Specifically, find the nozzle and throat areas and the mass flow through the system necessary to start and operate the system. At operating conditions, sketch the Mach number distribution along the wind tunnel from the nozzle inlet to just past the diffuser throat.

Design Problem #2

A fixed-geometry, convergent-divergent wind tunnel diffuser is to be designed for Mach number 2. Assuming no friction, compute the minimum possible percent losses in stagnation pressure during operation for the following cases:

(a) The best possible design is employed.

(b) The design is conservative, with a throat area 5 percent larger than that required for starting, and with the shock located during operation at an area 5 percent greater than the throat area.

(c) The converging portion is eliminated, and the process comprises a normal shock followed by reversible subsonic compression.

Design Problem #3

A range is to fly at 12,000 m altitude with a speed of 3200 km/hr.

(a) Design the best fixed-geometry, convergent-divergent diffuser for this aircraft, and compute for it the least percent loss in stagnation pressure.

(b) Suppose that it were possible to overdesign the aircraft to 3400 km/hr. Design the best convergent-divergent diffuser which could then be used, and find for it the least percent loss in stagnation pressure at the operating speed of 3200 km/hr.

Design Problem #4

Consider the supersonic wind tunnel which is to have an operating section Mach number of $M = 2$ and a circular test cross-section of approximately 10 cm in diameter.

(a) As a zero-order iteration, you decide to start out by matching a simple converging-diverging conical nozzle to get some idea for what's required:

The nozzle is equipped with a static pressure hole in the exit plane. The air for the system is supplied from a large reservoir at a pressure of 10 bar and room temperature of 20 °C (293 K). The supersonic nozzle (and eventually the wind tunnel) will exhaust into a large partially evacuated tank at a pressure in the range of approximately 0.1 bar. For the above conical nozzle under the specified conditions, pitot static-pressures measurements in the test section near the exit plane indicate pressures of $P_{\text{pist}} = 108.67$ bar and $P_{\text{static}} = 0.9083$ bar. Determine the exit Mach number and the nozzle efficiency.

(b) Based on these preliminary tests, you now are in a position to machine out the conical nozzle to a final converging-diverging supersonic de Laval nozzle, as shown here:

For the same intake reservoir and exhaust tank conditions, you now find that the pitot-static pressure measurements in the exit plane give a pressure ratio of 1.5.

Design Problem #5

A supersonic aircraft is equipped with a two-dimensional, converging-diverging, variable throat area, intake diffuser. The diffuser is designed for a cruise Mach number of 2.2. What percent increase in throat area is required to "swallow" the shock? If in the takeoff comprises the aircraft has to "feed" at Mach 1.8 due to tactical reasons, what percent of mass supply of air occurs, with the rest, of course, passing through the cowl, if the diffuser configuration happens to be set for cruise at $M_c = 2.2$?

Design Problem #6

Consider a continuous supersonic wind tunnel as illustrated below:

The test section has a cross-sectional area of 5 m², and the wind tunnel should be designed such that the pressure in the test section is 0.1 atm and the Mach number in the test section is 1.5. Perform the following tasks:

(a) For a fixed-geometry nozzle and a fixed-geometry diffuser, find the nozzle throat area ($A_t$) and the diffuser throat area ($A_d$), and sketch the Mach number distribution between stations 1 and 6.

(b) For a fixed-geometry nozzle and a variable-geometry diffuser, find the nozzle throat area and the minimum and maximum diffuser throat area. As well, sketch the Mach number and pressure distribution between stations 1 and 6 when the wind tunnel operates at maximum efficiency.

(c) What role does the compressor play for a fixed-geometry diffuser? What role does the compressor play for a variable-geometry diffuser?

Design Problem #7

Consider a supersonic wind tunnel. When the tunnel is operating in normal conditions, a shock exists at the throat of the diffuser as follows:

Knowing that the pressure ratio across this shock is of 1.8, and knowing that the throat area of the diffuser needs to be increased by a factor of 1.5 when starting the tunnel (i.e. $A_d$ from $A_{d,min} = 1.5A_{d,max}$), find the Mach number in the test section under normal operating conditions.

Answers

1. 6.2 km/s, 68 cm², 137 cm².
2. 27.9%.
3. (c) 3.14, $\mu_{\text{air}} = 0.9846$ (b) 2.0, $\mu_{\text{air}} = 0.5287$ (d) 0.666 cm², 1.77.
4. 102.2 km, 294 K.
5. 59.2%, 41.7%.
6. 3.1.

Due on Tuesday October 30th at 10:30. Do Problems 2 and 5 only.