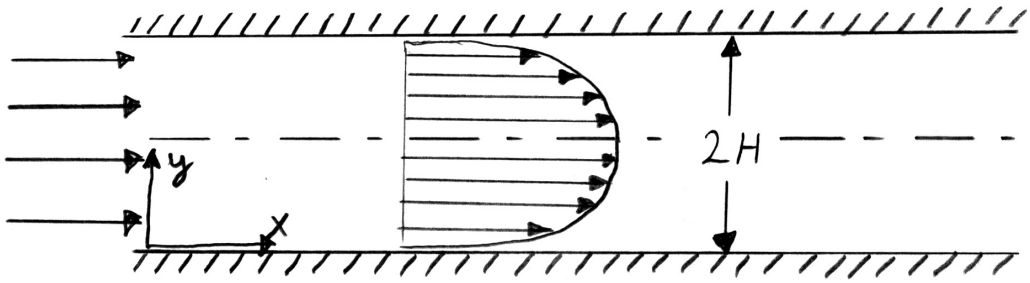
Viscous Flow Assignment 7 — Computational Viscous Flow

Consider the following two-dimensional channel with smooth walls:



Part #1

Starting from the Navier-Stokes equations, demonstrate that the x-momentum equation for laminar fully-developed flow in a channel collapses to:

$$\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) = \frac{\partial P}{\partial x}$$

Part #2

Starting from the x-momentum equation derived in part #1, demonstrate that the exact solution to this problem for constant μ and constant ρ is of:

$$u = -\frac{3\mu \mathrm{Re}_{D_H}}{\rho D_H H^2}\left(\frac{y^2}{2} - Hy\right)$$

with

$$\mathrm{Re}_{D_H} = -\frac{\rho D_H H^2}{3\mu^2}\frac{\partial P}{\partial x}$$

Part #3

Using the finite-volume approach, derive a set of discretization equations for the physical model outlined in Part 1 and for a 1D grid with uniform grid spacing. Also write down the discretization equations in coefficient form for both inner nodes and boundary nodes.

Part #4

Using the C code template attached and with $H = 0.1$ m, $\rho = 1000$ kg/m^3, $\mu = 10^{-3}$ kg/ms, and $dP/dx = -2$ Pa/m, do the following:

(a) Modify the function "find_coefficients_and_rhs" so that it uses the coefficients derived in Part #3 with N the number of nodes:

```
/* finds the discretization coefficients a,b,c in kg/m3s and the R
HS rhs in Pa/m for all nodes given
   N -> number of nodes
   H -> half height of the channel in m
   mu -> viscosity of the fluid in kg/ms
   dPdx -> pressure gradient along x in Pa/m
 */
void find_coefficients_and_rhs(long N, double H, double mu, double
dPdx, double *a, double *b, double *c, double *rhs){


}
```

EDIT Viscous_Flow_A7.c

(b) Compare graphically the solution obtained to the exact solution in Part #2 for the number of nodes N set to 5, 10, and 50 nodes. Are you surprised at the results?

(c) Print out the C code you used to obtain results.

assign7_template.c