Viscous Flow Assignment 8 — Turbulence Modeling

Question #1

Starting from the Reynolds-averaged constant-density momentum equation

$$\rho\left(\frac{\partial \bar{v}_i}{\partial t}+\sum_j \frac{\partial \bar{v}_i \bar{v}_j}{\partial x_j}\right) = -\frac{\partial \bar{P}}{\partial x_i}+\sum_j \frac{\partial \overline{\tau_{ji}}}{\partial x_j} - \sum_j \rho \frac{\partial \overline{v_i' v_j'}}{\partial x_j}$$

Derive the modeled form of the momentum equation (through the Boussinesq approximation):

$$\rho\left(\frac{\partial \bar{u}}{\partial t}+\sum_j \frac{\partial \bar{v}_i \bar{v}_j}{\partial x_j}\right) = -\frac{\partial \bar{P}}{\partial x_i}+\sum_j \frac{\partial}{\partial x_j}(\mu+\mu_t)\left(\frac{\partial \bar{v}_i}{\partial x_j}+\frac{\partial \bar{v}_j}{\partial x_i}\right)$$

with the turbulence eddy viscosity defined as:

$$\mu_t \equiv \frac{1}{2}\rho V_t L_t$$

Question #2

Starting from the definition of the turbulence eddy viscosity:

$$\mu_t \equiv \frac{1}{2}\rho V_t L_t$$

Explain as well as possible the Prandtl mixing length model for a flow over a flat plate:

$$\mu_t \approx \rho L_m^2 \left|\frac{\partial u}{\partial y}\right|$$

with

$$L_m = 0.435 y\left(1-\exp(-y^+/26)\right)$$

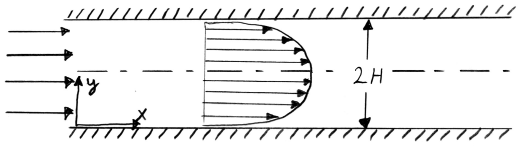
and with

$$y+ \equiv \frac{y\sqrt{\rho \tau_w}}{\mu}$$

with τ_w the shear stress at the wall.

Question #3

Consider the following two-dimensional channel with smooth walls:



Modify the function "find_coefficients_and_rhs" within the attached C code that solves the Prandtl algebraic turbulence model using a TDMA algorithm. Then, do the following:

(a) Compare the bulk velocity obtained numerically to the one obtained through the Jones correlation. If you implemented the algorithm well, both should be within 20% of each other.

(b) Find N that yields a "grid-converged" solution. To do so, start with $N = 100$, and then keep on doubling N until u_b obtained numerically does not vary by more than 1% compared to its value on the coarser grid. What is the minimum value of N that achieves this?

(c) Plot the velocity along y for N found in part (b).

Use $H = 0.1$ m, $\rho = 1000\ \mathrm{kg/m^3}$, $\mu = 10^{-3}$ kg/ms, and $dP/dx = -2$ Pa/m.

EDIT Viscous_Flow_A8.c

assign8_template.c

Notes

The Jones correlation applicable within the range $2000 \le \mathrm{Re}_{D_H} \le 2\times 10^5$ corresponds to:

$$\frac{1}{\sqrt{f_D}} = 2.0\log_{10}\left[\frac{2}{3}\mathrm{Re}_{D_H}\sqrt{f_D}\right]-0.8$$

where H is the half height of the channel, D_H the hydraulic diameter and f_D is the Darcy friction factor:

$$f_D = -\frac{dP}{dx}D_H \Big/ \left(\frac{1}{2}\rho u_b^2\right)$$

where u_b can be written as a function of the Reynolds number as follows:

$$\mathrm{Re}_{D_H} = \frac{\rho u_b D_H}{\mu}$$

Answers

3. The correct bulk velocity obtained with the Prandtl model lies between 0.25 and 0.28 m/s.