2018 Numerical Analysis Midterm Exam

Wednesday October 31st 2018
16:30 — 18:30

NO NOTES OR BOOKS; ANSWER ALL 4 QUESTIONS; ALL QUESTIONS HAVE EQUAL VALUE.

Question #1

(a) Consider a number of real type. Knowing that the machine accuracy (non-denormal) is of $\epsilon_{\text{mach}} = 9.5367 \times 10^{-7}$ and that the maximum positive number must be at least as high as 10^{23}, do the following:
(i) find the minimum number of bits for the exponent;
(ii) find the minimum number of bits for the significand.

(b) Consider the number 9.5367×10^{-4} stored in memory as a real type. Knowing that the exponent of the real type has 4 bits what is the minimum number of bits that the significand should have if the relative error on the number is less than 0.01?

Question #2

Consider the following system of equations:

$$-3x_1 + 2x_2 + 4x_3 + 5x_4 = 1$$
$$-x_1 + 3x_2 - x_3 + 5x_4 = 2$$
$$-4x_1 + 4x_3 + 2x_4 = 3$$
$$6x_2 + 4x_3 - 5x_4 = 4$$

Find x_1, x_2, x_3, x_4 using Gaussian elimination in two different ways:

(a) By hand.

(b) By writing a C program that starts as follows:

```
#include <stdio.h>
#include <stdlib.h>
#include <assert.h>

/* set number of rows to a constant */
#define N 4

int main(void){
  double A[N][N], B[N], X[N], Aorig[N][N], Borig[N];
  long row,row2,col;
  double fact,sum;
```

EDIT Numerical_Analysis_M2018Q2.c

Question #3

The secant method (a.k.a. Newton-Raphson) can be written in the following form:

$$x_{n+1} = x_n - \frac{f_n}{(f'_n)_{\text{secant}}}$$

where n is the iteration count, $f_n = f(x_n)$, and

$$(f'_n)_{\text{secant}} = \frac{f_n - f_{n-1}}{x_n - x_{n-1}}$$

Do the following:

(a) Using Taylor series expansion, prove that $(f'_n)_{\text{secant}}$ is first-order accurate. That is, prove that

$$(f'_n)_{\text{secant}} = f'_n + O(\Delta x)$$

where $O(\Delta x)$ stands for a sum of terms where the largest term scales with $x_n - x_{n-1}$.

(b) You wish to improve the secant method by finding an approximation for f' that is second order accurate. That is, derive using the Taylor series an expression for $(f'_n)_{\text{secant2}}$ such that

$$(f'_n)_{\text{secant2}} = f'_n + O(\Delta x^2)$$

Question #4

You wish to solve a system of equations $AX = B$ given the square matrix A and the vector B. Derive an expression for the work needed to find X when using Gaussian elimination *without* pivoting. The work needed should be written as a function of N (i.e., the number of rows within A) and should be simplified as much as possible. Explain clearly all steps needed to determine the work. Twenty points will be given for a clear explanation of how the work is determined, and five points for the correct answer. Note: the lower and upper triangular matrices of A are not known.