2018 Numerical Analysis Midterm Exam

Wednesday October 31st 2018
16:30 — 18:30

NO NOTES OR BOOKS; ANSWER ALL 4 QUESTIONS; ALL QUESTIONS HAVE EQUAL VALUE.

Question #1

(a) Consider a number of real type. Knowing that the machine accuracy (non-denormal) is of $\epsilon_{\rm mach}=9.5367\times10^{-7}$ and that the maximum positive number must be at least as high as 10^{23}, do the following:
(i) find the minimum number of bits for the exponent;
(ii) find the minimum number of bits for the significand.

(b) Consider the number 9.5367×10^{-4} stored in memory as a real type. Knowing that the exponent of the real type has 4 bits what is the minimum number of bits that the significand should have if the relative error on the number is less than 0.01?

Question #2

Consider the following system of equations:

$$-3x_1+2x_2+4x_3+5x_4=1$$
$$-x_1+3x_2-x_3+5x_4=2$$
$$-4x_1+4x_3+2x_4=3$$
$$6x_2+4x_3-5x_4=4$$

Find x_1, x_2, x_3, x_4 using Gaussian elimination in two different ways:

(a) By hand.

(b) By writing a C program that starts as follows:

```
#include <stdio.h>
#include <stdlib.h>
#include <assert.h>

/* set number of rows to a constant */
#define N 4

int main(void){
  double A[N][N], B[N], X[N], Aorig[N][N], Borig[N];
  long row,row2,col;
  double fact,sum;
```

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Question #3

The secant method (a.k.a. Newton-Raphson) can be written in the following form:

$$x_{n+1}=x_n-\frac{f_n}{(f_n')_{\rm secant}}$$

where n is the iteration count, $f_n=f(x_n)$, and

$$(f_n')_{\rm secant}=\frac{f_n-f_{n-1}}{x_n-x_{n-1}}$$

Do the following:

(a) Using Taylor series expansion, prove that $(f_n')_{\rm secant}$ is first-order accurate. That is, prove that

$$(f_n')_{\rm secant}=f_n'+O(\Delta x)$$

where $O(\Delta x)$ stands for a sum of terms where the largest term scales with x_n-x_{n-1}.

(b) You wish to improve the secant method by finding an approximation for f' that is second order accurate. That is, derive using the Taylor series an expression for $(f_n')_{\rm secant2}$ such that

$$(f_n')_{\rm secant2}=f_n'+O(\Delta x^2)$$

Question #4

You wish to solve a system of equations $AX=B$ given the square matrix A and the vector B. Derive an expression for the work needed to find X when using Gaussian elimination *without* pivoting. The work needed should be written as a function of N (i.e., the number of rows within A) and should be simplified as much as possible. Explain clearly all steps needed to determine the work. Twenty points will be given for a clear explanation of how the work is determined, and five points for the correct answer. Note: the lower and upper triangular matrices of A are not known.