Numerical Analysis Questions & Answers

Question by Student 201427127

Professor, I can't distinct what is prod1 and prod2 at C++ programing code that you made which was π and $\sum$ at your note on blackboard. Thank you. I can use LATEX now.

It was written Π — not π — on the blackboard. I explained prod1 and prod2 again at the beginning of the class today. 0.5 point bonus for the effort.

Question by Student 201527121

I want to prove it by solving simple equation.

$$y = \sqrt{(g^2+1)} \pm \ldots.$$

substitue small terms into x

$$y = \sqrt{(g^2+1)} + x$$

$$\sqrt{(g^2+1)} + x = \sqrt{(g^2+1 \pm 2\epsilon_{mach} g^2)}$$

Square both sides.

$$(g^2+1) + 2\sqrt{g^2+1}x + x^2 = (g^2+1 \pm 2\epsilon_{mach} g^2)$$

$$2\sqrt{g^2+1}x = -(g^2+1) + (g^2+1) \pm 2\epsilon_{mach} g^2 - x^2$$

Square of both sides again

$$4(g^2+1)x^2 = x^4 \mp 4\epsilon_{mach} g^2 x^2 + 4\epsilon_{mach}^2 g^4$$

ϵ_{mach}^2 *is enough small to assume zero value. Rearrange function and eliminate* x^2 *and* ϵ_{mach}^2

$$4(g^2+1) \pm 4\epsilon_{mach} g^2 = x^2$$

$$x = \pm 2\sqrt{g^2+1 \pm \epsilon_{mach} g^2}$$

As a result, we can guess

$$y = \sqrt{g^2+1} \pm 2\sqrt{g^2+1 \pm \epsilon_{mach} g^2}$$

There is a problem with your proof. When you write

$$4(g^2+1)x^2 = x^4 \mp 4\epsilon_{mach} g^2 x^2 + 4\epsilon_{mach}^2 g^4$$

You cannot say that the term $\epsilon_{mach}^2 g^4$ is negligible because $\epsilon_{mach}^2 \ll \epsilon_{mach}$. Here, g^4 can be much greater than g^2.. Thus, $\epsilon^2 g^4$ can be as large or larger than $\epsilon g^2 x^2$ and the final answer you give for x is wrong. Nonetheless I'll give you 2 points bonus for the effort.

Question by Student 201427148

Professor, I want to check my Answer of the problem, which handed-out at Class 10th, Sep 2018.

$$-GivenEquation.\, y = \sqrt{g^2 + 2\epsilon_{MACH} g^2 + 1}$$

$$-WantedEquation.\, y = \sqrt{g^2 \pm 1} + x$$

$$-UsedEquation.\, (a+b)^c = a^c + bca^{c-1} + \ldots +$$

$$-PROBLEM : Findx.$$

1st, I found that The Equation which must be used to solve the problem(Used Equation) is the Binomial Expansion. So, I Transform the Used Equation to Binomial Expansion like under Equation.

$$(a+b)^c = a^c + c\left(\frac{1}{1!}\right) a^{c-1} b + c(c-1)\left(\frac{1}{2!}\right) a^{c-2} b^2 + For(a \gg b)$$

Then I Transform the Given Equation like under Equation.

$$y = \sqrt{(g^2+1) \pm \epsilon_{MACH} g^2}$$

And, Permutate each term to a, b, c.

$$a = g^2 + 1,$$

$$b = \pm 2\epsilon_{MACH} g^2,$$

$$c = \frac{1}{2}$$

Before substituting the Permutated terms to Expansion, I confirmed that the condition $(a \gg b)$ is satisfied. As g is larger number and ϵ_{MACH} is almost zero. Then, a is more bigger than g and b is almost zero. So, I could use the upper Binomial Expansion. Then, I Substituted Each terms to Expanded Equation like under.

$$y = \sqrt{(g^2+1) \pm \epsilon_{MACH} g^2}$$

$$= \sqrt{g^2+1} \pm 2\epsilon_{MACH}\left(\frac{1}{2}\right)\left(\frac{1}{1!}\right)\frac{1}{\sqrt{g^2+1}}$$

$$+ (\pm 2\epsilon_{MACH} g^2)^2 \left(\frac{1}{2}\right)\left(\frac{-1}{2}\right)\left(\frac{1}{2!}\right)(\sqrt{g^2+1})^{\frac{-2}{3}} + \ldots +$$

In this Expanded Series, I found the pattern. The 'n'th terms *has* $(\epsilon_{MACH})^{n-1}$. *As* $(\epsilon_{MACH})^2$ *is almost zero,that I could ignore all terms without 1st term* $\left(\sqrt{g^2+1}\right)$ *and 2nd term* $\left(\pm 2\epsilon_{MACH}\left(\frac{1}{2}\right)\left(\frac{1}{1!}\right)\frac{1}{\sqrt{g^2+1}}\right)$. *Finally Two Terms left from Series.*

$$y \approx \sqrt{g^2+1} \pm 2\epsilon_{MACH}\left(\frac{1}{2}\right)\left(\frac{1}{1!}\right)\frac{1}{\sqrt{g^2+1}}$$

Thus, the wanted x *is*

$$x = \pm\frac{g^2}{\sqrt{g^2+1}}\epsilon_{MACH}$$

I'll wait your advice for my first using LATEX and my solution. Thank you.

That's not a bad proof, but it will fail to be valid if $(\epsilon_{\text{mach}} g^2)^2$ is not much smaller than ϵ_{mach}. For $\epsilon_{\text{mach}} \approx 10^{-15}$, this will happen when $g > 6000$.. Thus, your proof is valid only for not so high values of g. I'll give you 2 points bonus for the effort.

Question by Student 201727142

Professor, I can't understand that you teached last class about ϵ_x. *You teached* $X = 1.0/(SQRT(g^2+1)+g)$ *and you said* $y = \sqrt{g^2+1} \pm \epsilon_{mach}\sqrt{g^2+1}$. *and then i think X=1.0/(y+g) ,* $X = \frac{1\pm\epsilon_{mach}}{\sqrt{g^2+1}\pm\epsilon_{mach}\sqrt{g^2+1}+g}$. *But, you teached us* $X = \frac{1\pm\epsilon_{mach}}{\sqrt{g^2+1}\pm\epsilon_{mach}\sqrt{g^2+1}}$. *Why is (denominator's g) disappeared?*

Oups, I lost the g on the denominator. So, keep g there and add $\epsilon_{\text{mach}} g$ on the denominator. Then, you'll get (in worse case scenario):

$$x = \frac{1 \pm \epsilon_{\text{mach}}}{\sqrt{g^2+1} + g \pm \epsilon_{\text{mach}}(\sqrt{g^2+1} + |g|)}$$

You can figure out the following steps on your own following the same logic as shown in class.. Good observation: 2 points bonus.

Question by Student 201427113

Dear prof. B. Parent When i wanted to change base 2 :0000 0000 to base 10, the answer is 1 in the note during class. But i can't understand why the answer is 1 and it's two' complement is also 1. Because in case of one's complement, same base 2 0000 0000 is base10 '0' so i'm confused.

00000000 is zero either with the two's complement or the one's complement. I think you mean 11111111. Then, the one's complement is -0 but the two's complement is -1. You may have the rows mixed up.

Question by Student 201527119

Professor, I have an question about Question #6 in Assignment 01.
I think Question #6 i) *and* ii) *are changed, and answer is wrong too.*
Therefore, i) *is significand. and* ii) *is exponent.*

The answers given for one question are not necessarily in the correct order. There is no mistake in the answers. Think about this more carefully.