Numerical Analysis Questions & Answers

Question by Student 201427148

Professor, I want to check my Answer of the problem, which handed-out at Class 10th, Sep 2018.

$-Given Equation. y = \sqrt{g^2 + 2\epsilon_{MACH} g^2 + 1}$

$-Wanted Equation. y = \sqrt{g^2 \pm 1} + x$

$-Used Equation. (a+b)^c = a^c + bca^{c-1} + \ldots +$

$-PROBLEM : Find x.$

1st, I found that The Equation which must be used to solve the problem(Used Equation) is the Binomial Expansion. So, I Transform the Used Equation to Binomial Expansion like under Equation.

$$(a+b)^c = a^c + c\left(\frac{1}{1!}\right)a^{c-1}b + c(c-1)\left(\frac{1}{2!}\right)a^{c-2}b^2 + For(a \gg b)$$

Then I Transform the Given Equation like under Equation.

$$y = \sqrt{(g^2+1) \pm \epsilon_{MACH} g^2}$$

And, Permutate each term to a, b, c.

$a = g^2 + 1,$

$b = \pm 2\epsilon_{MACH} g^2,$

$c = \frac{1}{2}$

Before substituting the Permutated terms to Expansion, I confirmed that the condition $(a \gg b)$ is satisfied. As g is larger number and ϵ_{MACH} is almost zero. Then, a is more bigger than g and b is almost zero. So, I could use the upper Binomial Expansion. Then, I Substituted Each terms to Expanded Equation like under.

$$y = \sqrt{(g^2+1) \pm \epsilon_{MACH} g^2}$$

$$= \sqrt{g^2+1} \pm 2\epsilon_{MACH}\left(\frac{1}{2}\right)\left(\frac{1}{1!}\right)\frac{1}{\sqrt{g^2+1}}$$

$$+ \left(\pm 2\epsilon_{MACH} g^2\right)^2 \left(\frac{1}{2}\right)\left(\frac{-1}{2}\right)\left(\frac{1}{2!}\right)(\sqrt{g^2+1})^{\frac{-2}{3}} + \ldots +$$

In this Expanded Series, I found the pattern. The 'n'th terms has $(\epsilon_{MACH})^{n-1}$. As $(\epsilon_{MACH})^2$ is almost zero,that I could ignore all terms without

1st term $\left(\sqrt{g^2+1}\right)$ *and 2nd term* $\left(\pm 2\epsilon_{MACH}\left(\frac{1}{2}\right)\left(\frac{1}{1!}\right)\frac{1}{\sqrt{g^2+1}}\right)$. *Finally Two Terms left from Series.*

$$y \approx \sqrt{g^2+1} \pm 2\epsilon_{MACH}\left(\frac{1}{2}\right)\left(\frac{1}{1!}\right)\frac{1}{\sqrt{g^2+1}}$$

Thus, the wanted x *is*

$$x = \pm\frac{g^2}{\sqrt{g^2+1}}\epsilon_{MACH}$$

I'll wait your advice for my first using LATEX and my solution. Thank you.

That's not a bad proof, but it will fail to be valid if $(\epsilon_{\text{mach}} g^2)^2$ is not much smaller than ϵ_{mach}. For $\epsilon_{\text{mach}} \approx 10^{-15}$, this will happen when $g > 6000$.. Thus, your proof is valid only for not so high values of g. I'll give you 2 points bonus for the effort.

Question by Student 201727142

Professor, I can't understand that you teached last class about ϵ_x. *You teached* $X = 1.0/(SQRT(g^2+1)+g)$ *and you said* $y = \sqrt{g^2+1} \pm \epsilon_{mach}\sqrt{g^2+1}$. *and then i think X=1.0/(y+g)* , $X = \frac{1 \pm \epsilon_{mach}}{\sqrt{g^2+1} \pm \epsilon_{mach}\sqrt{g^2+1}+g}$. *But, you teached us* $X = \frac{1 \pm \epsilon_{mach}}{\sqrt{g^2+1} \pm \epsilon_{mach}\sqrt{g^2+1}}$. *Why is (denominator's g) disappeared?*

Oups, I lost the g on the denominator. So, keep g there and add $\epsilon_{\text{mach}} g$ on the denominator. Then, you'll get (in worse case scenario):

$$x = \frac{1 \pm \epsilon_{\text{mach}}}{\sqrt{g^2+1} + g \pm \epsilon_{\text{mach}}(\sqrt{g^2+1} + |g|)}$$

You can figure out the following steps on your own following the same logic as shown in class.. Good observation: 2 points bonus.

Question by Student 201427113

Dear prof. B. Parent When i wanted to change base 2 :0000 0000 to base 10, the answer is 1 in the note during class. But i can't understand why the answer is 1 and it's two' complement is also 1. Because in case of one's complement, same base 2 0000 0000 is base10 '0' so i'm confused.

00000000 is zero either with the two's complement or the one's complement. I think you mean 11111111. Then, the one's complement is -0 but the two's complement is -1. You may have the rows mixed up.

Question by Student 201527119

Professor, I have an question about Question #6 in Assignment 01.

I think Question $\#6$ i) and ii) are changed, and answer is wrong too. Therefore, i) is significand. and ii) is exponent.

The answers given for one question are not necessarily in the correct order. There is no mistake in the answers. Think about this more carefully.

Question by Student 201527147

Dear professor, I wonder if the range lower limit should be min pos normal or min pos denormal in question 5. Thank you.

If it's not indicated, then choose the lowest possible.

Question by Jaehyuk

Professor, I have question about Machine Precision of Float. For denormal number, $(0.f)_{min} = 0.00...0$(23'0's)$=0$. Then, $\varepsilon_{mach} = \frac{0.00...01-0}{0.00...01} = \frac{2^{-23}}{2^{-23}} = 1$. Is this the right ε_{mach} for float and denormal number?

Yes, you're on the right track. But $(0.f)_{\text{min}} = 0.00..001$, not 0. Thus, if $(0.f) = (0.f)_{\text{min}}$, then $\epsilon_{\text{mach}} = 1$. But ϵ_{mach} is not always 1 for denormal numbers. In fact, it is much less than 1 as $(0.f)$ becomes larger. 2 points bonus.