

Numerical Analysis Questions & Answers

Question by Student 201527119

*Professor, I have an question about Question #6 in Assignment 01.
I think Question #6 i) and ii) are changed, and answer is wrong too.
Therefore, i) is significand. and ii) is exponent.*

The answers given for one question are not necessarily in the correct order. There is no mistake in the answers. Think about this more carefully.

Question by Student 201527147

Dear professor, I wonder if the range lower limit should be min pos normal or min pos denormal in question 5. Thank you.

If it's not indicated, then choose the lowest possible.

Question by Jaehyuk

Professor, I have question about Machine Precision of Float. For denormal number, $(0.f)_{\min} = 0.00\dots 0(23'0's)=0$. Then, $\epsilon_{\text{mach}} = \frac{0.00\dots 01-0}{0.00\dots 01} = \frac{2^{-23}}{2^{-23}} = 1$. Is this the right ϵ_{mach} for float and denormal number?

Yes, you're on the right track. But $(0.f)_{\min} = 0.00\dots 001$, not 0. Thus, if $(0.f) = (0.f)_{\min}$, then $\epsilon_{\text{mach}} = 1$. But ϵ_{mach} is not always 1 for denormal numbers. In fact, it is much less than 1 as $(0.f)$ becomes larger. 2 points bonus.

Question by Student 201427127

*Professor, I'll prove the question that you told us at the class before the lecture.
 $A < n \dots \neg$ and $B < n + 1 \dots \neg$. And then, I'll subtract \neg from \neg . the result of this is $A - B < -1$. But if A is equal to B or A is bigger than B , this inequation will be false because left handside is positive number or zero. The prove is over.*

It's fine, but it's not clear enough. You need to provide a more clear example. 1.5 point bonus. Also, avoid naming equations with Hangul. See the L^AT_EX mini HOWTO in the Skylounge on how to give a number to equations.

Question by Student 201427113

Professor, I'll prove the question about inequation that you told us on class.

$A < n \dots \neg$ and $B < n + 1 \dots \neg$ And then, I'll subtract \neg from \neg . the result of this is $A - B < -1$. For example, if $A = n - 1$ and $B = n - 1$ so

$A - B = 0 < -1$ is false. So for generally A is equal to B or A is bigger than B , this inequation will be false because left handside is positive number or zero and right handside is -1 so although general form is inconsistent. The prove is over.

Better. 0.5 point extra bonus. If you would have typeset this with proper L^AT_EX equation numbers you would gotten more.

Question by Student 201527121

1. wanted Prove

$$\frac{1}{1+y} = 1 - y + y^2 - y^3 + \dots$$

2. Given condition

$$|y| \ll 1$$

3. Solution

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots = \sum_{n=1}^{\infty} (x)^n$$

Let's substitute

$$x = -y$$

$$\frac{1}{1+y} = 1 - y + y^2 - y^3 + \dots$$

4. Let's prove

$$\frac{1}{1-x} = \sum_{n=1}^{\infty} (x)^n$$

$$|x| \ll 1 \text{ (It should be for convergence)}$$

$$S_n = \sum_{k=1}^n (x)^k$$

$$S_n = x + x^2 + x^3 + \dots + x^n$$

$$(1-x)S_n = (1-x)(x+x^2+x^3+\dots+x^n) \\ = x+x^2+\dots+x^n - x^2-x^3-\dots-x^{n+1} = x-x^{n+1}$$

$$S_n = \frac{x}{1-x} - \frac{x^{n+1}}{1-x}$$

Because of $|x| < 1$, If $n \rightarrow \infty$ then $S_n \rightarrow \frac{x}{1-x}$

$$\sum_{k=N}^n (x)^k = x^N + \dots + x^n = x^{N-1} \sum_{k=1}^{n-N+1} (x)^k$$

$$\sum_{k=N}^{\infty} x^k = \lim_{\rightarrow} (n \rightarrow \infty) \sum_{k=N}^n x^k = \lim_{\rightarrow} (n \rightarrow \infty) \cdot x^{N-1} \sum_{k=1}^{n-N+1} x^k = \frac{x^N}{1-x}$$

If

$$N = 0$$

then

$$\sum_{k=0}^{\infty} x^k = \frac{1}{1-x}$$

5. Conclusion

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots = \sum_{n=1}^{\infty} (x)^n$$

$$x = -y$$

$$\frac{1}{1-x} = \sum_{n=1}^{\infty} (x)^n$$

$$\frac{1}{1+y} = 1 - y + y^2 - y^3 + \dots$$

This is great. I'll give you 3 points bonus. It could be made a bit shorter thus, and you shouldn't separate the proof into sections: this makes it harder to read. Also, I was looking for a simpler proof using Taylor series. If someone else can do it, another 3 bonus points are up for grabs.