Numerical Analysis Questions & Answers

Question by Student 201427127

Professor, I'll prove the question that you told us at the class before the lecture. $A < n\ldots$ ㄱ and $B < n+1\ldots$ ㄴ And then, I'll substract ㄴ from ㄱ. the result of this is $A - B < -1$. But if A is equal to B or A is bigger than B, this inequation will be false because left handside is positive number or zero. The prove is over.

It's fine, but it's not clear enough. You need to provide a more clear example. 1.5 point bonus. Also, avoid naming equations with Hangul. See the LaTeX mini HOWTO in the Skylounge on how to give a number to equations.

Question by Student 201427113

Professor, I'll prove the question about inequation that you told us on class. $A < n\ldots$ ㄱ and $B < n+1\ldots$ ㄴ And then, I'll substract ㄴ from ㄱ. the result of this is $A - B < -1$. For example, if $A = n - 1 and B = n - 1$ so $A - B = 0 < -1$ is false. So for generally A is equal to B or A is bigger than B, this inequation will be false because left handside is positive number or zero and right handside is -1 so although general form is inconsistent. The prove is over.

Better. 0.5 point extra bonus. If you would have typeset this with proper LaTeX equation numbers you would gotten more.

Question by Student 201527121

1. wanted Prove

$$\frac{1}{1+y} = 1 - y + y^2 - y^3 + \ldots$$

2. Given condition

$|y| << 1$

3. Solution

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots = \sum_{n=1}^{\infty}(x)^n$$

Let's substitute

$$x = -y$$

$$\frac{1}{1+y} = 1 - y + y^2 - y^3 + \ldots$$

4. Let's prove

$$\frac{1}{1-x} = \sum_{n=1}^{\infty}(x)^n$$

$|x| << 1$ *(It should be for convergence)*

$$S_n = \sum_{k=1}^{n}(x)^k$$

$$S_n = x + x^2 + x^3 + \ldots + x^n$$

$$(1-x)S_n = (1-x)(x + x^2 + x^3 + \ldots + x^n)$$

$$= x + x^2 + \ldots + x^n - x^2 - x^3 - \ldots - x^{n+1} = x - x^{n+1}$$

$$S_n = \frac{x}{1-x} - \frac{x^{n+1}}{1-x}$$

Because of $|x| << 1$, *If* $n \to \infty$ *then* $S_n \to \frac{x}{1-x}$

$$\sum_{k=N}^{n}(x)^k = x^N + \ldots + x^n = x^{N-1}\sum_{k=1}^{n-N+1}(x)^k$$

$$\sum_{k=N}^{\infty} x^k = \underset{\longrightarrow}{\lim}(n \to \infty)\sum_{k=N}^{n} x^k = \underset{\longrightarrow}{\lim}(n \to \infty) \cdot x^{N-1}\sum_{k=1}^{n-N+1} x^k = \frac{x^N}{1-x}$$

If

$$N = 0$$

then

$$\sum_{k=0}^{\infty} x^k = \frac{1}{1-x}$$

5.*Conclusion*

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots = \sum_{n=1}^{\infty}(x)^n$$

$$x = -y$$

$$\frac{1}{1-x} = \sum_{n=1}^{\infty}(x)^n$$

$$\frac{1}{1+y} = 1 - y + y^2 - y^3 + \ldots$$

This is great. I'll give you 3 points bonus. It could be made a bit shorter thus, and you shouldn't separate the proof into sections: this makes it harder to read. Also, I was looking for a simpler proof using Taylor series. If someone else can do it, another 3 bonus points are up for grabs.

Question by Student 201427113

Professor, I'll prove $\frac{1}{1+x} = 1 - x + x^2 - x^3 \ldots$
Taylor series is one of the power series.
So $f(x) = C_0 + C_1(x-a) + C_2(x-a)^2 + C_3(x-a)^3 + \ldots . \rightarrow A - equation$
And we have to decide what is $C_0, C_1, C_2, C_3, \ldots ..$
1. Substitute x=a on A-equation
So $f(a) = C_0$
2. Apply differential on A-equation
So $f'(x) = C_1 + 2C_2(x-a) + 3C_3(x-a)^2 + \ldots . \rightarrow B - equation$
And substitute x=a on B-equation
So $f'(a) = C_1$
3. Apply differential on B-equation So
$f''(x) = 2C_2 + 6C_3(x-a) + 12C_4(x-a)^2 \ldots . \rightarrow C - equation$
And substitute x=a on C-equation
So $f''(a) = 2C_2 \rightarrow C_2 = \frac{f''(a)}{2}$
4. Apply differential on C-equation So
$f'''(x) = 6C_3 + 24C_4(x-a) \ldots . \rightarrow D - equation$
And substitute x=a on D-equation
So $f'''(a) = 6C_3 \rightarrow C_3 = \frac{f'''(a)}{6}$
5. Apply this sequence infinitely and generalized the eqs.
$C_n = \frac{f^n(a)}{n!}$
Finally substitute $C_n = \frac{f^n(a)}{n!}$ *on A-equation*
So
$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \ldots$
$= \sum_{n=0}^{\infty} \frac{f^n(a)}{n!}(x-a)^n$

$f(x) = \frac{1}{1+x} = (1+x)^{-1} \rightarrow P - equation$
$f'(x) = -(1+x)^{-2} \rightarrow Q - equation$
$f''(x) = 2(1+x)^{-3} \rightarrow R - equation$
$f'''(x) = -6(1+x)^{-4} \rightarrow S - equation$

Apply x=a=0 on eqs. P,Q,R,S

$f(0) = 1$
$f'(0) = -1$
$f''(0) = 2$
$f'''(0) = -6$

So $f(x) = 1 - 1 \times x + \frac{2}{2!}x^2 + \frac{-6}{3!}x^3 + \ldots$

Prove is finished.!!

This is fine, I'll give you 2 points bonus. I would have given 3 if you would have typeset correctly your equations with proper equation numbers as specified in the LaTeX mini HOWTO in the Skylounge. But, this proof is still not what I was looking for.. There's a much easier way to prove it using Taylor series (without power series). Three bonus points are still up for grabs.

Question by Student 201527119

Proof.

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \ldots$$

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!}f"(a) + \frac{(x-a)^3}{3!}f"'(a) + \ldots$$

In

$$\frac{1}{1+x}$$

a=0
So,

$$f(x) = f(0) + (x-0)f'(0) + \frac{(x-0)^2}{2!}f"(0) + \frac{(x-0)^3}{3!}f"'(0) + \ldots$$

$$f'(x) = \frac{-1}{(1+x)^2}, f"(x) = \frac{2}{(1+x)^3}, f"'(x) = \frac{-6}{(1+x)^4}$$

$$f(0) = 1, f'(0) = -1, f"(0) = 2, f"'(0) = -6$$

Put them in equation.

$$f(x) = \frac{1}{1+x} = 1 - x + x^2 - x^3 + \ldots$$

End.

This is what I was looking for. I'll give you 2.5 points bonus. The only problem here is that you don't define $f(x)$ prior to doing the expansion with the Taylor series.

Question by Jaehyuk

Dear professor, I have question about A2#Q6(a). According to the question, iterative method is $x^{n+1} = x^n - 0.05 * \frac{f(x^n)}{f\prime(x^n)} - 0.95 * \frac{f(x^n)(x^{n-1} - x^{n-2})}{f(x^{n-1}) - f(x^{n-2})}$ *However, in order to use this as an iterative method, isn\`t this should be like this;*
$x_{n+1} = x_n - 0.05 * \frac{f(x_n)}{f\prime(x_n)} - 0.95 * \frac{f(x_n)(x_{n-1} - x_{n-2})}{f(x_{n-1}) - f(x_{n-2})}$

Well, in this case, it's clear what the notation means. It doesn't matter if it's a superscript or subscript..