Numerical Analysis Questions & Answers

Question by Student 201527119

Proof.

$$\frac{\epsilon_{(n+1)(Newton)}}{\epsilon_{(n+1)(secant)}} = |\epsilon_0 * k|^{2^n - 1.618^n}$$

$$k = |\frac{f"(r)}{2f'(r)}|$$

$$\epsilon_{(n+1)(Newton)} = k\epsilon^2$$

$$\epsilon_1 = k * \epsilon_0^2$$

$$\epsilon_2 = k * \epsilon_1^2 = k^3 * \epsilon_0^4$$

$$\epsilon_3 = k * \epsilon_2^2 = k^7 * \epsilon_0^8$$

$$...$$

$$\epsilon_n = k^{2^n - 1} * \epsilon_0^{2^n}$$

$$\epsilon_{(n+1)(secant)} = k^{\frac{1}{1.168}} \epsilon^{1.168}$$

(1.168=p)

$$\epsilon_1 = k^{p^{-1}} * \epsilon_0^p$$

$$\epsilon_2 = k^{p^{-1}} * \epsilon_1^p = k^{1+p^{-1}} * \epsilon_0^{p^2}$$

$$\epsilon_3 = k^{p^{-1}} * \epsilon_2^p = k^{p+1+p^{-1}} * \epsilon_0^{p^3}$$

$$...$$

$$\epsilon_n = k^{\sum_{m=0}^{n-1} p^{m-1}} * \epsilon_0^{p^n}$$

Consequently,

$$\frac{\epsilon_{n(newton)}}{\epsilon_{n(secant)}} = \frac{k^{2^n - 1} * \epsilon_0^{2^n}}{k^{\sum_{m=0}^{n-1} p^{m-1}} * \epsilon_0^{p^n}} = \frac{k^{2^n} * \epsilon_0^{2^n}}{k * k^{\sum_{m=0}^{n-1} p^{m-1}} * \epsilon_0^{p^n}}$$

$$k^{\sum_{m=0}^{n-1} p^{m-1}} = k^{1+\sum_{m=0}^{n-1} p^{m-1}}$$

$$1+\sum_{m=0}^{n-1} p^{m-1} = 1+p^{-1}+1+p+\ldots+p^{n-2}$$

Using Geometric sequence sum,

$$p^{-1}+1+p+\ldots+p^{n-2} = \frac{p^{n}-1}{p^{2}-p}$$

$$p^{2} = p+1$$

$$So, \frac{p^{n}-1}{p^{2}-p} = p^{n}-1$$

=>

$$\frac{k^{2^n} * \epsilon_0^{2^n}}{k * k^{p^n-1} * \epsilon_0^{p^n}} = \frac{k^{2^n} * \epsilon_0^{2^n}}{k^{p^n} * \epsilon_0^{p^n}}$$

$$Consequently, |\epsilon_0 * k|^{2^n-1.618^n}$$

$$So, \frac{\epsilon(n)(Newton)}{\epsilon(n)(secant)} = |\epsilon_0 * k|^{2^n-1.618^n}$$

$$\frac{\epsilon(n)(Newton)}{\epsilon(n)(secant)} = |\epsilon_0 * \frac{f"(r)}{2f'(r)}|^{2^n-1.618^n}$$

I don’t understand this part:

$$k^{\sum_{m=0}^{n-1} p^{m-1}} = k^{1+\sum_{m=0}^{n-1} p^{m-1}}$$

This doesn’t make sense to me. Is this a typo or a misunderstanding? Please fix this and retype your proof below.

Question by Student 201527119

Sorry, I missed k in left.

$$k * k^{\sum_{m=0}^{n-1} p^{m-1}} = k^{1+\sum_{m=0}^{n-1} p^{m-1}}$$

Hm, but if that is true, then I don’t understand how you obtain the following on the denominator on the LHS:

$$\frac{k^{2^n} * \epsilon_0^{2^n}}{k * k^{p^n-1} * \epsilon_0^{p^n}} = \frac{k^{2^n} * \epsilon_0^{2^n}}{k^{p^n} * \epsilon_0^{p^n}}$$

Please fix this and rewrite your proof better below. And please never use the newline command in your posts (two blackslashes). This is not necessary and is a bad habit.

Question by Student 201727142

Hello, professor. I can't understand that the bisection method in Assignment 2 #1. If the significant number is 4, should I round to the fourth decimal places? or round down to the fourth decimal place? And do I have to count 1 when I get the x_{mid} *in the initial range*$(x_{min} = \frac{\pi}{2} \leqq x \leqq x_{max} = \frac{\pi*3}{2})$*?*

Obtaining convergence to 4 significant numbers means that iterating more won't change the first 4 significant numbers. Only one question per post please. 1 point bonus.

Question by Student 201527119

$$\epsilon_{(n)(Newton)} = k^{2^n-1}\epsilon_0^{2n}$$

$$\epsilon_{(n)(secant)} = k^{\sum_{m=0}^{n-1} p^{m-1}}\epsilon_0^{p^n}$$

$$\frac{\epsilon_{(n)(Newton)}}{\epsilon_{(n)(secant)}} = \frac{k^{2^n-1}\epsilon_0^{2n}}{k^{\sum_{m=0}^{n-1} p^{m-1}}\epsilon_0^{p^n}} = \frac{\frac{k^{2^n}\epsilon_0^{2n}}{k}}{k^{\sum_{m=0}^{n-1} p^{m-1}}\epsilon_0^{p^n}} = \frac{k^{2^n}\epsilon_0^{2n}}{k^1 k^{\sum_{m=0}^{n-1} p^{m-1}}\epsilon_0^{p^n}} = \frac{k^{2^n}\epsilon_0^{2n}}{k^{1+\sum_{m=0}^{n-1} p^{m-1}}\epsilon_0^{p^n}}$$

$$(k^a k^b = k^{a+b})$$

In denominator,

$$k^{\sum_{m=0}^{n-1} p^{m-1}}\epsilon_0^{p^n} = k^{1+p^{-1}+p^0+p^1+\ldots+p^{n-2}}\epsilon_0^{p^n}$$

$$p^{-1} + p^0 + p^1 + \ldots + p^{n-2} = \frac{\frac{p^n-1}{p}}{p-1} = \frac{p^n - 1}{p^2 - p}$$

$$and, p^2 = p + 1$$

$$so, \frac{p^n - 1}{p^2 - p} = p^n - 1$$

$$k^{\sum_{m=0}^{n-1} p^{m-1}}\epsilon_0^{p^n} = k^{1+p^{-1}+p^0+p^1+\ldots+p^{n-2}}\epsilon_0^{p^n} = k^{1+p^n-1}\epsilon_0^{p^n} = k^{p^n}\epsilon_0^{p^n}$$

$$\frac{\epsilon_{(n)(Newton)}}{\epsilon_{(n)(secant)}} = \frac{k^{2n}\epsilon_0^{2n}}{k^{p^n}\epsilon_0^{p^n}} = \frac{(k\epsilon_0)^{2^n}}{(k\epsilon_0)^{p^n}} = (k\epsilon_0)^{2^n-p^n}$$

$$So, p = 1.618, k = |\frac{f"(r)}{2f'(r)}|$$

$$\frac{\epsilon_{(n)(Newton)}}{\epsilon_{(n)(secant)}} = (|\frac{f"(r)}{2f'(r)}|\epsilon_0)^{2^n-1.618^n}$$

I think this is much better than before one.

There is still the same problem with your proof. You claim that

$$\frac{\epsilon(n)(Newton)}{\epsilon(n)(secant)} = \frac{k^{2^n-1}\epsilon_0^{2n}}{k^{\sum_{m=0}^{n-1}p^{m-1}}\epsilon_0^{p^n}} = \frac{\frac{k^{2^n}\epsilon_0^{2n}}{k}}{k^{\sum_{m=0}^{n-1}p^{m-1}}\epsilon_0^{p^n}} = \frac{k^{2^n}\epsilon_0^{2n}}{k^1 k^{\sum_{m=0}^{n-1}p^{m-1}}\epsilon_0^{p^n}} = \frac{k^{2^n}\epsilon_0^{2n}}{k^{1+\sum_{m=0}^{n-1}p^{m-1}}\epsilon_0^{p^n}}$$

and that

$$k^{\sum_{m=0}^{n-1}p^{m-1}}\epsilon_0^{p^n} = k^{1+p^{-1}+p^0+p^1+\ldots+p^{n-2}}\epsilon_0^{p^n} = k^{1+p^n-1}\epsilon_0^{p^n} = k^{p^n}\epsilon_0^{p^n}$$

But if you substitute the latter in the former, you will not get

$$\frac{\epsilon(n)(Newton)}{\epsilon(n)(secant)} = \frac{k^{2n}\epsilon_0^{2n}}{k^{p^n}\epsilon_0^{p^n}} = \frac{(k\epsilon_0)^{2^n}}{(k\epsilon_0)^{p^n}} = (k\epsilon_0)^{2^n-p^n}$$

Thus your proof is wrong. I gave you 3 chances already, and you didn't succeed. I can not give you one more chance. 1 point bonus for the effort. Let others now try.

Question by Student 201727142

Hello, professor. I'm so sorry to have asked a lot of questions. But I still don't understand the second question. And do I have to count 1 when I get the x_{mid} *in the initial range(*$x_{min} = \frac{\pi}{2} \leqq x \leqq x_{max} = \frac{\pi * 3}{2}$*)?*

I don't understand your question. Please formulate this better.

Question by Student 201427128

Dear professor, I have question about A2#Q6(a).

before that, when you solved problem using eq.Newton-Raphson by hand in the class.

$x_{n+1} = x_n - \frac{f(x_n)(x_n - x_{n-1})}{x_n - x_{n-1}}$
$f(x) = x^2 - 2$
you guessed $x_0 = 1,\ x_{-1} = 0.9$ *and found* $x_1 = 1.526$
And with many iterations, can get a more accurate x.

I used this idea to solve A2#Q6(a).
$x_{n+1} = x_n - 0.05\frac{f(x_n)}{f'(x_n)} - 0.95\frac{f(x_n)(x_{n-1} - x_{n-2})}{f(x_{n-1}) - f(x_{n-2})}$
given initial condition $x_0 = 2.8$
I guessed $x_1 = 3.0,\ x_2 = 3.2$
And I got the answers(=5 iterations) on the site.

but now I have question that if I guess more bigger x1 x2 , I get another iteration number
Is there a more accurate way to guess?

I'm not sure what you mean. The number of iterations depends on the initial guess. If the initial guess is the root, then the number of iterations is 1. I'll give you 0.5 point for the effort.