Numerical Analysis Questions & Answers

Question by Jaehyuk

Professor, I have question on A#4Q3-(b). According to the definition of the order of convergence, we can derive p from $|\epsilon_{n+1}| = k \times |\epsilon_n|^p$. *Here starts my problem. That is, for* x_1, *the root is -0.0025, the 1st guess is 0, the 1st iteration yields 5, and the 2nd iteration yields 4.91244. Assigning these values into the formula above, this ends up as follows;* $|5 - (-0.025)| = k \times |0 - (-0.025)|^p$ *and* $|4.91244 - (-0.025)| = k \times |5 - (-0.025)|^p$. *This yields* $k = 4.96393$ *and* $p = -0.003315$ *which does not match with the answer. I am wondering which part of the process is wrong.*

You're not doing anything wrong. If you would answer this in the exam, you would get full points. But, in general, we do not know what the exact root is. Thus, it is necessary to approximate it as the solution at the next iteration. If using the solution at the next iteration (i.e. 4.573738E+00) as the root, you'll get the answer listed. I made this more clear within the question formulation. 2 points bonus.

Question by Jaehyuk

Professor, I have question about the number of arithmetic operation of Lagrage Polynomial. For example, with 4 data points,
$p_3(x) = \frac{(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)} + \frac{(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_1)(x_2-x_3)(x_2-x_4)} + \frac{(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_1)(x_3-x_2)(x_3-x_4)}$ *. Then*
$+ \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_1)(x_4-x_2)(x_4-x_3)}$
change the order of deniminator as follows;
$p_3(x) = -\frac{(x-x_2)(x-x_3)(x-x_4)}{(x_2-x_1)(x_3-x_1)(x_4-x_1)} + \frac{(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_1)(x_3-x_2)(x_4-x_2)} - \frac{(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_1)(x_3-x_2)(x_4-x_3)}$ *. By doing*
$+ \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_1)(x_4-x_2)(x_4-x_3)}$
so, I can reduce the number of subtraction of denominator;
$(x_4 - x_3), (x_4 - x_2), (x_4 - x_1), (x_3 - x_2), (x_3 - x_1), (x_2 - x_1)$. *This can reduce the number of arithmetic operation to find and save the value of denominator from 20(3subtraction and 2multiplication for 4 terms*$((3 + 2) \times 4)$*) to 14(2multiplication for 4terms and 6 subtration*$(2 \times 4 + 6)$*). Am I on the right track to reduce the number of the operation?*

I don't understand well your question. Why are there only 3 terms within $p_3(x)$ and not 4? Also, the rest of the question doesn't make much sense to me. You need to explain this better. 0.5 point bonus for the effort. Make sure to use the PREVIEW button and check if the question looks as intended. Also, use $$ and

not $ for long math expressions.

Question by Jaehyuk

Professor, I have question about A#6 Reminder. In the last row which includes b_{i+1}*, the range of i is* $2 \leqq i \leqq (N-1)$*. As far as I know, however,if i goes up to (N-1), then this would create* b_N*. This means there are N equations not (N-1). I am confusing why there are N equations for cubic splines.*

Good question. We need to find b_N because a_{N-1} and c_{N-1} depend on b_N. So, we only need to find $N-1$ intervals, but we need to find N bs. 2 points bonus.

Question by Student 201527105

Professor, i have a question about Piecewise linear interpolation. I wonder if Piecewise liner interpolation's function has always straight grape line in each intervals. This interpolation is simple to use, but it seems to be bad when viewed from the side of derivative. In each intervals, derivatives may be discontinuous. And i also think that this is a little bad method to estimate arbitrary values in the given intervals. What is the big difference of this method when compared to other interpolations.

Hm, I'm not sure what confuses you. Answers to your questions were given in class.

Question by Student 201427113

Professor, When you explain Analytical matrix immersion, Work time about Inverse matrix for 2X2
$A=$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$A^{-1} = 1/det(A) \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

$det(A) = a_{11}a_{22} - a_{12}a_{21}$
$W_{det(A)} = 3$ *("2" multi. and "1" sub.)*
$W_{A^{-1}} = W_{det(A)} + 4Div. = 3 + 4 = 7$
But When Change marix

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

to

$$\begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

Why Work time is not apllied in matrix change.

There's little work involved when moving numbers around in memory compared with additions or multiplications. Hence why it's not counted. 1 point bonus.

Question by Student 201527105

professor, i have a question about Assignment7-Q#4. To calculate the error, the actual value of intergal should be known. According to the trapezoidal rule and the simpson rule, i obtained appoximate values. But i don't know the actual value of given integral. How can i get it? / And i also wonder if the error always decreases as N increases. (Intuitively, i think it is true.)

You can find the exact value by using a very large N. So, just double N until the value of the integral doesn't change significantly anymore and use this as the exact value. 2 points bonus.