Numerical Analysis Assignment 3 — Systems of Equations

Question #1

Consider the following system of equations:

$$
\begin{aligned}
-3x_1 + 2x_2 + 4x_3 + 5x_4 &= 1 \\
-x_1 + 3x_2 - x_3 + 5x_4 &= 2 \\
-4x_1 + 4x_3 + 2x_4 &= 3 \\
6x_2 + 4x_3 - 5x_4 &= 4
\end{aligned}
$$

Find x_1, x_2, x_3, x_4 using Gaussian elimination in two different ways:

(a) By hand.

(b) By writing a C program that starts as follows:

```
#include <stdio.h>
#include <stdlib.h>
#include <assert.h>

/* set number of rows to a constant */
#define N 4

int main(void){
  double A[N][N], B[N], X[N], Aorig[N][N], Borig[N];
  long row,row2,col;
  double fact,sum;
```

EDIT Numerical_Analysis_A3Q1.c

Question #2

Consider a system of equations expressed as:

$AX = B$

with

$$
A = \begin{bmatrix}
-2 & 0 & 0 & 0 \\
0 & 3 & -1 & 5 \\
-4 & 0 & 4 & 2 \\
0 & 6 & -2 & -5
\end{bmatrix}
$$

Find the matrices L and U by hand such that $A = LU$ and L is a lower-triangular matrix and U is a upper-triangular matrix.

Question #3

Consider the system of equations

$AX = B$

with

$$B = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

and with A as outlined in Question #2. Using the L and U matrices found by hand in Question #2 solve for X using lower-upper decomposition in two ways:

(a) By hand

(b) Using a C program that starts as follows:

```
#include <stdio.h>
#include <stdlib.h>
#include <assert.h>

/* set number of rows to 4 */
#define N 4

int main(void){
  double RHS, L[N][N], U[N][N], B[N], Xprime[N], X[N];
  long row,col;
```

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Question #4

For the following system of equations

$AX = B$

do the following:

(a) Prove that the work needed in making the Gaussian elimination augmented matrix M upper triangular is

$$W_{\text{ut}} = \sum_{n=1}^{N-1} \sum_{m=n+1}^{N} (2(N-n)+3)$$

Note that M has a size of $(N+1) \times N$ because it is composed of A and B. Explain carefully all the logical steps involved in proving the latter.

(b) Should $A = LU$ with L lower triangular and U upper triangular and for L and U given, prove that the work needed in finding X corresponds to:

$$W_{LU} = \sum_{n=1}^{N} (4(N-n)+2)$$

Explain carefully all the logical steps involved in proving the latter.

Due on Monday October 29th at 16:30. Do all 4 problems. Note that this assignment is more difficult than average and will be given 6 points bonus instead of 3.