

Numerical Analysis Assignment 3 — Systems of Equations

Due on Monday October 29th at 16:30. Do all 4 problems. Note that this assignment is more difficult than average and will be given 6 points bonus instead of 3.

Question #1

Consider the following system of equations:

$$\begin{aligned} -3x_1 + 2x_2 + 4x_3 + 5x_4 &= 1 \\ -x_1 + 3x_2 - x_3 + 5x_4 &= 2 \\ -4x_1 + 4x_3 + 2x_4 &= 3 \\ 6x_2 + 4x_3 - 5x_4 &= 4 \end{aligned}$$

Find x_1, x_2, x_3, x_4 using Gaussian elimination in two different ways:

- (a) By hand.
(b) By writing a C program that starts as follows:

```
#include <stdio.h>
#include <stdlib.h>
#include <assert.h>

/* set number of rows to a constant */
#define N 4

int main(void) {
    double A[N][N], B[N], X[N], Aorig[N][N], Borig[N];
    long row, row2, col;
    double fact, sum;

```

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Question #2

Consider a system of equations expressed as:

$$AX = B$$

with

$$A = \begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & 3 & -1 & 5 \\ -4 & 0 & 4 & 2 \\ 0 & 6 & -2 & -5 \end{bmatrix}$$

Find the matrices L and U by hand such that $A = LU$ and L is a lower-triangular matrix and U is an upper-triangular matrix.

Question #3

Consider the system of equations

$$AX = B$$

with

$$B = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

and with A as outlined in Question #2. Using the L and U matrices found by hand in Question #2 solve for X using lower-upper decomposition in two ways:

- (a) By hand
(b) Using a C program that starts as follows:

```
#include <stdio.h>
#include <stdlib.h>
#include <assert.h>

/* set number of rows to 4 */
#define N 4

int main(void) {
    double RHS, L[N][N], U[N][N], B[N], Xprime[N], X[N];
    long row, col;

```

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Question #4

For the following system of equations

$$AX = B$$

do the following:

- (a) Prove that the work needed in making the Gaussian elimination augmented matrix M upper triangular is

$$W_{\text{ut}} = \sum_{n=1}^{N-1} \sum_{m=n+1}^N (2(N-n) + 3)$$

Note that M has a size of $(N+1) \times N$ because it is composed of A and B . Explain carefully all the logical steps involved in proving the latter.

- (b) Should $A = LU$ with L lower triangular and U upper triangular and for L and U given, prove that the work needed in finding X corresponds to:

$$W_{LU} = \sum_{n=1}^N (4(N-n) + 2)$$

Explain carefully all the logical steps involved in proving the latter.