Numerical Analysis Assignment 4 — Partial Pivoting and Non-Linear Systems

Question #1

Consider the system of equations $AX = B$ with A equal to:

$$A = \begin{bmatrix} -2 & 2 & -1 \\ 6 & -6 & 7 \\ 3 & -8 & 4 \end{bmatrix}$$

and B equal to:

$$B = \begin{bmatrix} -1 \\ -7 \\ -6 \end{bmatrix}$$

Find X using partial pivoting (by hand).

Question #2

Consider the following non-linear system of equations:

$x_1^4 + x_2 = 5$

$x_1 x_2 + x_2^{1.5} = 8$

Solve the latter using Newton's method using the initial conditions $x_1 = 1$ and $x_2 = 1$. Do it in two ways:

(a) by hand and solve the first 3 iterations only.

(b) By writing a C code that starts as follows:

```
#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include <assert.h>

double f(double x1, double x2){
  return(x1*x1*x1*x1+x2-5.0);
}

double dfdx1(double x1, double x2){
```

EDIT Numerical_Analysis_A4Q2.c

Question #3

Consider the following non-linear system of equations:

$x_2 x_1 x_3 = 5$

$\frac{1}{2}x_1^2 + \frac{1}{2}x_2^2 = 100$

$x_2 + x_3 = 0$

Do the following:

(a) Find x_1, x_2, and x_3 using Newton's method using the initial conditions $x_1 = 0$, $x_2 = 1$, and $x_3 = 1$. Do so by hand and solve the first 2 iterations only.

(b) Using the results obtained in (a) estimate the order of convergence of the method. Hint: the root should be found from the iterative procedure as the solution to the next iteration (valid as we are converging). Thus, the approximate root here would be 4.573738E+00, 2.731503E+01, -2.731503E+01.

Question #4

Consider the following non-linear system of equations:

$\sin^2(x_1)\cos(x_2) = 0.5$

$\sqrt{x_1} - x_2 = 0.3$

Do the following:

(a) Find the root of the system starting from the guess $x_1 = x_2 = 0.6$ and make sure the root is correct to at least 6 significant digits. Hint: first substitute one equation in the other to obtain 1 equation for 1 unknown, and then find the root for such unknown through an iterative root solver of your choice.

(b) Starting from the guess $x_1 = x_2 = 0.6$ and with the first steps $\Delta x_1 = \Delta x_2 = 0.00001$, use the secant method *in system form* to obtain the root of the system. Solve by hand the first 2 iterations only. Outline clearly all the steps including the expressions used to compute the Jacobians.

Question #5

Consider the system of equations $AX = B$ with A equal to:

$$A = \begin{bmatrix} -2 & 0 & 1 & 1 \\ 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 2 & 1 \end{bmatrix}$$

and B equal to:

$$B = \begin{bmatrix} -1 \\ -7 \\ 3 \\ -6 \end{bmatrix}$$

Using partial pivoting *only when the pivot is zero*, find the lower and upper triangular matrices associated with matrix A. Outline *all the steps needed* to obtain the matrix L, the matrix U, and the permutation matrices. Also, indicate clearly how A can be written as a function of L, U, and the permutation matrices.

Answers

1. $2, \frac{1}{4}, -\frac{5}{2}$.
2.
3. 4.91244, 51.125, -51.125, 0.1, -1.1, -1.18.
4. 0.919332, 0.658818, 0.9197, 0.6586.
5. $P_{34}LU$.

Due on Wednesday November 14th at 16:30. Do Questions #3, #4, and #5 only.