

# Numerical Analysis Assignment 4 — Partial Pivoting and Non-Linear Systems

## Question #1

Consider the system of equations  $AX = B$  with  $A$  equal to:

$$A = \begin{bmatrix} -2 & 2 & -1 \\ 6 & -6 & 7 \\ 3 & -8 & 4 \end{bmatrix}$$

and  $B$  equal to:

$$B = \begin{bmatrix} -1 \\ -7 \\ -6 \end{bmatrix}$$

Find  $X$  using partial pivoting (by hand).

## Question #2

Consider the following non-linear system of equations:

$$x_1^4 + x_2 = 5$$

$$x_1 x_2 + x_2^{1.5} = 8$$

Solve the latter using Newton's method using the initial conditions  $x_1 = 1$  and  $x_2 = 1$ . Do it in two ways:

- by hand and solve the first 3 iterations only.
- By writing a C code that starts as follows:

```
#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include <assert.h>

double f(double x1, double x2){
    return(x1*x1*x1*x1+x2-5.0);
}

double dfdx1(double x1, double x2){
    EDIT Numerical_Analysis_A4Q2.c
```

## Question #3

Consider the following non-linear system of equations:

$$x_2 x_1 x_3 = 5$$

$$\frac{1}{2}x_1^2 + \frac{1}{2}x_2^2 = 100$$

$$x_2 + x_3 = 0$$

Do the following:

- Find  $x_1$ ,  $x_2$ , and  $x_3$  using Newton's method using the initial conditions  $x_1 = 0$ ,  $x_2 = 1$ , and  $x_3 = 1$ . Do so by hand and solve the first 2 iterations only.
- Using the results obtained in (a) estimate the order of convergence of the method. Hint: the root should be found from the iterative procedure as the solution to the next iteration (valid as we are converging). Thus, the approximate root here would be 4.573738E+00, 2.731503E+01, -2.731503E+01.

## Question #4

Consider the following non-linear system of equations:

$$\sin^2(x_1) \cos(x_2) = 0.5$$

$$\sqrt{x_1} - x_2 = 0.3$$

Do the following:

- Find the root of the system starting from the guess  $x_1 = x_2 = 0.6$  and make sure the root is correct to at least 6 significant digits. Hint: first substitute one equation in the other to obtain 1 equation for 1 unknown, and then find the root for such unknown through an iterative root solver of your choice.
- Starting from the guess  $x_1 = x_2 = 0.6$  and with the first steps  $\Delta x_1 = \Delta x_2 = 0.00001$ , use the secant method *in system form* to obtain the root of the system. Solve by hand the first 2 iterations only. Outline clearly all the steps including the expressions used to compute the Jacobians.

## Question #5

Consider the system of equations  $AX = B$  with  $A$  equal to:

$$A = \begin{bmatrix} -2 & 0 & 1 & 1 \\ 2 & 1 & 0 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 0 & 2 & 1 \end{bmatrix}$$

and  $B$  equal to:

$$B = \begin{bmatrix} -1 \\ -7 \\ 3 \\ -6 \end{bmatrix}$$

Using partial pivoting *only when the pivot is zero*, find the lower and upper

triangular matrices associated with matrix  $A$ . Outline *all the steps needed* to obtain the matrix  $L$ , the matrix  $U$ , and the permutation matrices. Also, indicate clearly how  $A$  can be written as a function of  $L$ ,  $U$ , and the permutation matrices.

### Answers

1.  $2, \frac{1}{4}, -\frac{5}{2}$ .

2.

3.  $4.91244, 51.125, -51.125, 0.1, -1.1, -1.18$ .

4.  $0.919332, 0.658818, 0.9197, 0.6586$ .

5.  $P_{34}LU$ .

**Due on Wednesday November 14th at 16:30. Do Questions #3, #4, and #5 only.**