Numerical Analysis Assignment 6 — Piecewise Interpolation and Splines

Question #1

Consider the following set of data points:

x	y
0.1	0.03
0.3	0.06
0.8	0.07
1.1	0.1

Write a function f in C that returns y given x in the range $x_1 \leq x \leq x_4$ using piecewise linear interpolation:

```
#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include <assert.h>

double f(double x){
```

EDIT Numerical_Analysis_A6Q1.c

Question #2

Consider the following data points:

x	y
1	3
3	7
8	6
10	9
12	7
14	12

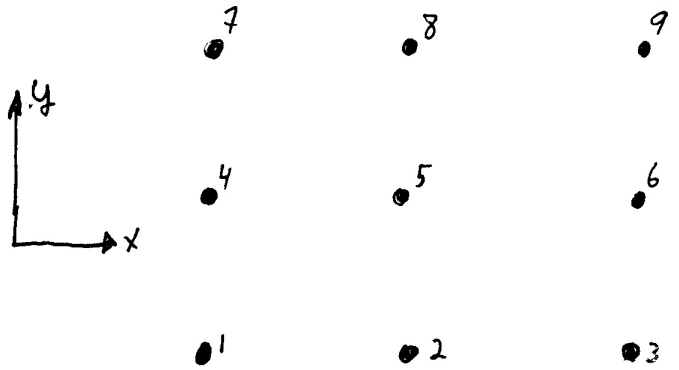
Do the following:

(a) Using a cubic spline, find the value (by hand) of y at $x = 6$. Derive proper boundary conditions and do basic verifications to ensure that your answer is correct.

(b) Using a Lagrange polynomial, find the value (by hand) of y at $x = 6$. Compare to the result obtained in (a) and discuss.

Question #3

Consider 9 nodes arranged as follows:



with the following values and x-y coordinates:

Node	x	y	ϕ
1	0	?	100
2	?	?	120
3	1	?	150
4	0	1	160
5	?	1	170
6	1	1	190
7	0	2	200
8	?	2	230
9	1	2	270

using a multidimensional piecewise-linear interpolation, it is found that:

$\phi_{x=0.2,y=1.6} = 190$

and

$\phi_{x=0.8,y=0.8} = 170$

Knowing that

$x_2 = x_5 = x_8$

$y_1 = y_2 = y_3$

find (in no particular order):

(a) The y coordinate of nodes 1, 2, and 3.

(b) The x coordinate of nodes 2, 5, and 8.

Question #4

Consider the following data points:

x	$f(x)$
1	2
2	4
4	3

It is given that at $x = 1$, $f''' = 0$. Using a cubic spline, find the value of $f(x)$ at $x = 3$. Specifically, do the following:

(a) Derive a boundary condition function of bs at the left boundary.

(b) Derive a boundary condition function of bs at the right boundary.

(c) Write down the equation for the center node function of bs.

(d) Solve the b equations in (a), (b), and (c) and evaluate f at $x = 3$.

(e) Perform basic verifications to ensure that your answer is correct.

Answers

3. -0.9, 0.733.

4. $\frac{13}{3}$.

Reminder

Equations for inner nodes within cubic splines:

$f_i(x) = a_i(x - x_i)^3 + b_i(x - x_i)^2 + c_i(x - x_i) + d_i$

$d_i = y_i$

$a_i = (b_{i+1} - b_i)/(3\Delta x_i) \text{ for } 1 \leq i \leq N - 1$

$c_i = \frac{\Delta y_i}{\Delta x_i} - b_i \Delta x_i - \left(\frac{b_{i+1} - b_i}{3} \right) \Delta x_i \text{ for } 1 \leq i \leq N - 1$

$\Delta x_{i-1} b_{i-1} + 2 \left(\Delta x_i + \Delta x_{i-1} \right) b_i + \Delta x_i b_{i+1} = 3 \left(\frac{\Delta y_i}{\Delta x_i} - \frac{\Delta y_{i-1}}{\Delta x_{i-1}} \right) \text{ for } 2 \leq i \leq N - 1$

Due on Monday 26 November at 16:30. Do Questions #1, #3, and #4 only.