Numerical Analysis Assignment 7 — Numerical Integration

Question #1

Using a previously-derived expression for the mid-point rule:

$$I_i = \Delta x_i f(x_m) + \frac{\Delta x_i^3}{24} f''(x_m) + \frac{\Delta x_i^5}{1920} f''''(x_m) + \ldots$$

Do the following:

(a) Show that the trapezoidal rule can be written as:

$$I_i = \frac{\Delta x_i}{2}(f(x_i) + f(x_i + \Delta x_i)) - \frac{\Delta x_i^3}{12} f''(x_m) - \frac{\Delta x_i^5}{480} f''''(x_m) + \ldots$$

(b) Show that the global error associated with the trapezoidal rule is $O(\Delta x^2)$

Question #2

Using the trapezoidal rule:

$$I_i = \frac{\Delta x_i}{2}(f(x_i) + f(x_i + \Delta x_i)) + O(\Delta x_i^3)$$

Write a C code that finds the numerical solution of the integral

$$\int_{x=1}^{x=2} e^{x^2} dx$$

with the number of integration steps N set to 50. The C code should start as follows:

```
#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include <assert.h>

double f(double x){
  double ret;
  ret=exp(x*x);
  return(ret);
}

int main(void){
```

EDIT Numerical_Analysis_A7Q2.c

Question #3

Making use of the Simpson rule:

$$I_i = \text{odd}(i)\frac{(\Delta x_i + \Delta x_{i+1})}{6}(f(x_i) + 4f(x_{i+1}) + f(x_{i+2})) + O(\Delta x_i^5)$$

Write a C code that finds the numerical solution of the integral

$$\int e^{x^2} dx$$

in the interval $1 \leq x \leq 2$ with the number of integration steps N set to 50. The C code should start as follows:

```
#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include <assert.h>

double f(double x){
  double ret;
  ret=exp(x*x);
  return(ret);
}

int main(void){
```

EDIT Numerical_Analysis_A7Q3.c

Note: the C code should give a high accuracy of the integral when N is odd and when N is even.

Question #4

For each of the C codes you developed for Questions #2 and #3, determine the error when integrating e^{x^2} in the interval $1 \leq x \leq 2$. For this purpose, tabulate the results in a table such as the following:

N	$\sum_i I_i$	$\left\lvert \sum_i I_i - \int_1^2 e^{x^2} dx \right\rvert$
3	..	..
7	..	..
15	..	..
31	..	..
4	..	..
8	..	..
16	..	..
32	..	..

Does the error (the last column) go down as expected? Discuss. Do this for

(a) The trapezoidal rule;

(b) The Simpson rule.

Question #5

You wish to create a new numerical integration method. To do so, you come up with the idea of evaluating the integral I_i by fitting a 2nd degree polynomial of the form

$$P_i(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2$$

through 3 data points within the ith interval. For this purpose, do the following:

(a) Express the polynomial coefficients a_i, b_i, and c_i as a function of the data points $(x_i,\ f_i)$, $(x_{i+1/2},\ f_{i+1/2})$, $(x_{i+1},\ f_{i+1})$.

(b) Using the polynomial coefficients derived in (a), find an expression for I_i over the interval $x_i \le x \le x_{i+1}$ and simplify as much as possible.

Answers

5. $b_i = \frac{2}{\Delta x_i}\left(2f_{i+1/2} - \frac{3}{2}f_i - \frac{1}{2}f_{i+1}\right)$.

Due on Monday December 10th at 16:30. Do Problems #1, #3, #4b, and #5 only.