Numerical Analysis Assignment 7 — Numerical Integration

Question #1

Using a previously-derived expression for the mid-point rule:

$$I_i = \Delta x_i f(x_m) + \frac{\Delta x_i^3}{24} f''(x_m) + \frac{\Delta x_i^5}{1920} f''''(x_m) + \ldots$$

Do the following:

(a) Show that the trapezoidal rule can be written as:

$$I_i = \frac{\Delta x_i}{2}(f(x_i) + f(x_i + \Delta x_i)) - \frac{\Delta x_i^3}{12} f''(x_m) - \frac{\Delta x_i^5}{480} f''''(x_m) + \ldots$$

(b) Show that the global error associated with the trapezoidal rule is $O(\Delta x^2)$

Question #2

Using the trapezoidal rule:

$$I_i = \frac{\Delta x_i}{2}(f(x_i) + f(x_i + \Delta x_i)) + O(\Delta x_i^3)$$

Write a C code that finds the numerical solution of the integral

$$\int_{x=1}^{x=2} e^{x^2}\, dx$$

with the number of integration steps N set to 50. The C code should start as follows:

```
#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include <assert.h>

double f(double x){
  double ret;
  ret=exp(x*x);
  return(ret);
}

int main(void){
```

EDIT Numerical_Analysis_A7Q2.c

Question #3

Making use of the Simpson rule:

$$I_i = \text{odd}(i)\frac{(\Delta x_i + \Delta x_{i+1})}{6}(f(x_i) + 4f(x_{i+1}) + f(x_{i+2})) + O(\Delta x_i^5)$$

Write a C code that finds the numerical solution of the integral

$$\int e^{x^2}\, dx$$

in the interval $1 \leq x \leq 2$ with the number of integration steps N set to 50. The C code should start as follows:

```
#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include <assert.h>

double f(double x){
  double ret;
  ret=exp(x*x);
  return(ret);
}

int main(void){
```

EDIT Numerical_Analysis_A7Q3.c

Note: the C code should give a high accuracy of the integral when N is odd and when N is even.

Question #4

Using the two C codes you developed for Questions #2 and #3, show the difference in accuracy between the Simpson rule and the Trapezoidal rule when integrating e^{x^2} in the interval $1 \leq x \leq 2$. For this purpose, tabulate the results in a table such as the following:

N	Method	$\sum_i I_i$	$\left\lvert \sum_i I_i - \int_1^2 e^{x^2} dx \right\rvert$
3	Trapezoidal	..	..
7	Trapezoidal	..	..
15	Trapezoidal	..	..
31	Trapezoidal	..	..
3	Simpson	..	..
7	Simpson	..	..
15	Simpson	..	..
31	Simpson	..	..

Does the error (the last column) go down as expected? Discuss.

Answers

1.
2.
3.
4.