

Numerical Analysis Assignment 8 —

Numerical Differentiation

Question #1

Consider an RC circuit for which the governing equation is:

$$R \frac{dq}{dt} + \frac{q}{C} = 0$$

Show that the exact solution is:

$$q(t) = q_0 \exp\left(-\frac{t}{RC}\right)$$

Question #2

Consider the modified Euler method of numerical differentiation. Do the following:

(a) Assuming that

$$\begin{aligned} & \Delta t f\left(t_{n+1/2}, \phi_n + \frac{\Delta t}{2} f(t_n, \phi_n) + O(\Delta t^2)\right) \\ &= \Delta t f\left(t_{n+1/2}, \phi_n + \frac{\Delta t}{2} f(t_n, \phi_n)\right) + O(\Delta t^3) \end{aligned}$$

Show that the modified Euler method is second-order accurate.

(b) For $f(t, \phi) = \frac{1}{\phi}$, show that the assumption in (a) is valid.

Question #3

Using the forward Euler method, solve q at $t = 1$ for the RC circuit equation

$$R \frac{dq}{dt} + \frac{q}{C} = 0$$

with $RC = 3$ and with the initial condition being $q_0 = 2$ at time $t = 0$ and with $\Delta t = 0.2$. Do so in two different ways:

(a) By hand

(b) With a C code that starts as follows:

```
#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include <assert.h>

#define dt 0.2
```

```
#define tmax 1.0
```

```
double f(
```

```
EDIT Numerical_Analysis_A8Q3.c
```

Note: your algorithm should make use of the defined dt and tmax and work for any value of dt or tmax.

Question #4

Using a second-order Runge-Kutta method, solve q at $t = 1$ for the RC circuit equation

$$R \frac{dq}{dt} + \frac{q}{C} = 0$$

with $RC = 3$, with the initial condition being $q_0 = 2$, with $\Delta t = 0.5$, and with the constraint $a = 0.5$. Do so in two different ways:

- (a) By hand
- (b) With a C code that starts as follows:

```
#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include <assert.h>
```

```
#define dt 0.5
#define tmax 1.0
```

```
double f(
```

```
EDIT Numerical_Analysis_A8Q4.c
```

Note: your algorithm should make use of the defined dt and tmax and work for any value of dt or tmax.

Recall

The 2nd order Runge-Kutta scheme can be expressed as

$$k_1 = \Delta t f(t_n, \phi_n)$$

$$k_2 = \Delta t f(t_n + \alpha \Delta t, \phi_n + \beta k_1)$$

$$\phi_{n+1} = \phi_n + a k_1 + b k_2$$

with the constraints

$$a + b = 1$$

$$b\alpha = \frac{1}{2}$$

$$b\beta = \frac{1}{2}$$

Due on Friday December 21st at 18:00. Do Questions #2, #3, and #4 only.