Numerical Analysis Assignment 8 — Numerical Differentiation

Question #1

Consider an RC circuit for which the governing equation is: $R \frac{dq}{dt}+\frac{q}{C}=0$ Show that the exact solution is: $q(t)=q_0 \exp\left(-\frac{t}{RC}\right)$

Question #2

Consider the modified Euler method of numerical differentiation. Do the following:

(a) Assuming that $\Delta t f\left(t_{n+1/2},-\phi_n+\frac{\Delta t}{2}f(t_n,-\phi_n) + O(\Delta t^2) \right) = \Delta t f\left(t_{n+1/2},-\phi_n+\frac{\Delta t}{2}f(t_n,-\phi_n) \right) + O(\Delta t^3)$ Show that the modified Euler method is second-order accurate.

(b) For $f(t,\phi)=\frac{1}{\phi}$, show that the assumption in (a) is valid.

Question #3

Using the forward Euler method, solve q at $t=1$ for the RC circuit equation $R \frac{dq}{dt}+\frac{q}{C}=0$ with $RC=3$ and with the initial condition being $q_0=2$ at time $t=0$ and with $\Delta t=0.2$. Do so in two different ways:

(a) By hand

(b) With a C code that starts as follows:

```
#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include <assert.h>

#define dt 0.2
#define tmax 1.0

double f(
```

EDIT Numerical_Analysis_A8Q3.c

Note: your algorithm should make use of the defined dt and tmax and work for any value of dt or tmax.

Question #4

Using a second-order Runge-Kutta method, solve q at $t=1$ for the RC circuit equation $R \frac{dq}{dt}+\frac{q}{C}=0$ with $RC=3$, with the initial condition being $q_0=2$, with $\Delta t=0.5$, and with the constraint $a=0.5$. Do so in two different ways:

(a) By hand

(b) With a C code that starts as follows:

```
#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include <assert.h>

#define dt 0.5
#define tmax 1.0

double f(
```

EDIT Numerical_Analysis_A8Q4.c

Note: your algorithm should make use of the defined dt and tmax and work for any value of dt or tmax.

Recall

The 2nd order Runge-Kutta scheme can be expressed as $k_1 = \Delta t f(t_n,-\phi_n) \\ k_2 = \Delta t f(t_n+\alpha \Delta t, - \phi_n + \beta k_1)\\ \phi_{n+1}=\phi_n + a k_1 + b k_2$ with the constraints $a+b=1\\ b\alpha=\frac{1}{2}\\ b\beta=\frac{1}{2}$

Due on Friday December 21st at 18:00. Do Questions #2, #3, and #4 only.