# Numerical Analysis Assignment 8 — Numerical Differentiation

4.

### Question #1

Consider an RC circuit for which the governing equation is:

$$R\frac{dq}{dt} + \frac{q}{C} = 0$$

Show that the exact solution is:

$$q(t) = q_0 \exp\left(-\frac{t}{RC}\right)$$

### Question #2

Consider the modified Euler method of numerical differentiation. Do the following:

(a) Assuming that

$$\Delta t f\left(t_{n+1/2},\ \phi_n + \frac{\Delta t}{2} f(t_n,\ \phi_n) + O(\Delta t^2)\right)$$
$$= \Delta t f\left(t_{n+1/2},\ \phi_n + \frac{\Delta t}{2} f(t_n,\ \phi_n)\right) + O(\Delta t^3)$$

Show that the modified Euler method is second-order accurate.

(b) For $f(t, \phi) = \frac{1}{\phi}$, show that the assumption in (a) is valid.

### Question #3

Using the forward Euler method, solve $q$ at $t = 1$ for the RC circuit equation

$$R\frac{dq}{dt} + \frac{q}{C} = 0$$

with $RC = 3$ and with the initial condition being $q_0 = 2$ at time $t = 0$ and with $\Delta t = 0.2$. Do so in two different ways:

(a) By hand

(b) With a C code that starts as follows:

```
#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include <assert.h>

#define dt 0.2
#define tmax 1.0

double f(
```

EDIT Numerical_Analysis_A8Q3.c

*Note:* your algorithm should make use of the defined dt and tmax and work for any value of dt or tmax.

### Question #4

Using a second-order Runge-Kutta method, solve $q$ at $t = 1$ for the RC circuit equation

$$R\frac{dq}{dt} + \frac{q}{C} = 0$$

with $RC = 3$, with the initial condition being $q_0 = 2$, with $\Delta t = 0.5$, and with the constraint $a = 0.5$. Do so in two different ways:

(a) By hand

(b) With a C code that starts as follows:

```
#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include <assert.h>

#define dt 0.5
#define tmax 1.0

double f(
```

EDIT Numerical_Analysis_A8Q4.c

*Note:* your algorithm should make use of the defined dt and tmax and work for any value of dt or tmax.

### Recall

The 2nd order Runge-Kutta scheme can be expressed as

$$k_1 = \Delta t f(t_n,\ \phi_n)$$
$$k_2 = \Delta t f(t_n + \alpha\Delta t,\ \phi_n + \beta k_1)$$
$$\phi_{n+1} = \phi_n + a k_1 + b k_2$$

with the constraints

$$a + b = 1$$
$$b\alpha = \frac{1}{2}$$
$$b\beta = \frac{1}{2}$$

### Answers

1.
2.
3.