

2016 Numerical Analysis Midterm Exam

When is the best time to have the midterm exam?

Tuesday Oct 25th 16:30 -- 18:30 [see note]	<input type="checkbox"/>	0
Thursday Oct 27th 16:30 -- 18:30 [see note]	<input type="checkbox"/>	6
Tuesday Nov 1st 16:30 -- 18:30 [see note]	<input type="checkbox"/>	4
Thursday Nov 3rd 16:30 -- 18:30	<input type="checkbox"/>	12
Friday Nov 4th 14:00 -- 16:00	<input type="checkbox"/>	1
Friday Nov 4th 16:30 -- 18:30	<input type="checkbox"/>	0
Friday Nov 4th 18:00 -- 20:00	<input type="checkbox"/>	1

Poll ended at 2:03 am on Tuesday October 18th 2016. Total votes: 24. Total voters: 17.

Thursday November 3rd 2016
16:30 — 18:30

NO NOTES OR BOOKS; ANSWER ALL 4 QUESTIONS; ALL QUESTIONS HAVE EQUAL VALUE.

Question #1

Consider the function $f = \sin(x)$ with x in radians. Find the root $f = 0$ for the initial interval $\frac{1}{2}\pi \leq x \leq \frac{3}{2}\pi$ using the bisection method. Do so in two different ways:

- (a) By hand, with enough iterations to yield a root accurate to at least 4 significant digits. How many iterations are needed to find a root accurate to at least 4 significant digits?
- (b) With a C code that starts as follows:

```
#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include <assert.h>

double f(double x){
    double ret;
    ret=sin(x);
    return(ret);
}
```

```
int main(void) {
```

Question #2

Consider the system of equations $AX = B$ with A equal to:

$$A = \begin{bmatrix} -2 & 2 & -1 \\ 6 & -6 & 7 \\ 3 & -8 & 4 \end{bmatrix}$$

and B equal to:

$$B = \begin{bmatrix} -1 \\ -7 \\ -6 \end{bmatrix}$$

Find X using partial pivoting (by hand).

Question #3

Consider a real number stored with 5 bytes. Bit #1 is reserved for the sign, while bits #2 to #10 are reserved for the biased exponent, and bits #11 to #40 are related to the significand. Do the following:

- (a) Find the minimum and maximum possible exponent p
- (b) Find the smallest possible positive number
- (c) Find the largest possible number
- (d) Find the smallest possible positive subnormal number

Question #4

Consider the following non-linear system of equations:

$$x_2 x_1 x_3 = 5$$

$$\frac{1}{2}x_1^2 + \frac{1}{2}x_2^2 = 100$$

$$x_2 + x_3 = 0$$

Do the following:

- (a) Find x_1 , x_2 , and x_3 using Newton's method using the initial conditions $x_1 = 0$, $x_2 = 1$, and $x_3 = 1$. Do so by hand and solve the first 2 iterations only.
- (b) Using the results obtained in (a) estimate the order of convergence of the method.

Answers

1. 13.

2. $2, \frac{1}{4}, -\frac{5}{2}$.

3. $255, -254, 3.454 \times 10^{-77}, 1.15792089 \times 10^{77}, 3.217 \times 10^{-86}$.

4. $4.91244, 51.125, -51.125, 0.1, -1.1, -1.18$.