2017 Computational Aerodynamics Midterm Exam

When is the best time for you?

Tue 18 Apr 16:30 -- 18:30	2
Thu 20 Apr 16:30 -- 18:30	6
Fri 21 Apr 13:00 -- 15:00	6
Tue 25 Apr 16:30 -- 18:30	2
Thu 27 Apr 16:30 -- 18:30	3
Fri 28 Apr 13:00 -- 15:00	0

Poll ended at 11:43 pm on Monday April 24th 2017. Total votes: 19. Total voters: 9.

Please choose your favourite time slots: you can choose up to 3 time slots and you can change your votes later if you wish. You must vote before Tuesday April 11 at 16:30.

Thursday April 20th 2017
18:00 — 20:00

NO NOTES OR BOOKS; USE CFD TABLES THAT WERE DISTRIBUTED; ANSWER ALL 4 QUESTIONS; ALL QUESTIONS HAVE EQUAL VALUE.

Question #1

Starting from the imposed dependencies on the generalized coordinates τ, ξ, and η:

Cartesian Coordinates	Generalized Coordinates
$t=t(\tau)$	$\tau=\tau(t)$
$x=x(\xi,\eta,\tau)$	$\xi=\xi(x,y,t)$
$y=y(\xi,\eta,\tau)$	$\eta=\eta(x,y,t)$

Demonstrate that the metrics of the generalized coordinates correspond to:

$$\xi_t=\frac{\Gamma}{\Omega}(y_\tau x_\eta - x_\tau y_\eta)\,,\quad \xi_x=\frac{y_\eta}{\Omega},\quad \xi_y=-\frac{x_\eta}{\Omega}$$

and

$$\eta_t = \frac{\Gamma}{\Omega}(x_\tau y_\xi - x_\xi y_\tau), \quad \eta_x = -\frac{y_\xi}{\Omega}, \quad \eta_y = \frac{x_\xi}{\Omega}$$

with $\Gamma \equiv \tau_t$ and Ω the inverse of the metrics Jacobian defined in 2D as:

$$\Omega \equiv x_\xi y_\eta - y_\xi x_\eta$$

Question #2

Starting from the principle of conservation of mass, show that the mass conservation equation for a fluid corresponds to:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0$$

with ρ the mass density, and u, v, w the x, y, z components of the velocity vector.

Question #3

Consider the following system of equations:

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} = 0$$

with

$$U = \begin{bmatrix} \rho_1 \\ \rho_2 \\ \rho u \\ \rho E \end{bmatrix} = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} \quad \text{and} \quad F = \begin{bmatrix} \rho_1 u \\ \rho_2 u \\ \rho u^2 + P \\ \rho u H \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}$$

with

$$E = \frac{\rho_1}{\rho} c_{v1} T + \frac{\rho_2}{\rho} c_{v2} T + \frac{u^2}{2}$$

$$H = \frac{\rho_1}{\rho} c_{p1} T + \frac{\rho_2}{\rho} c_{p2} T + \frac{u^2}{2}$$

$$P = (\rho_1 R_1 + \rho_2 R_2)\, T$$

$$\rho = \rho_1 + \rho_2$$

and with c_{v1}, c_{v2}, c_{p1}, c_{p2}, R_1, R_2 some constants. Using the method of your choice find the following elements within the flux Jacobian:

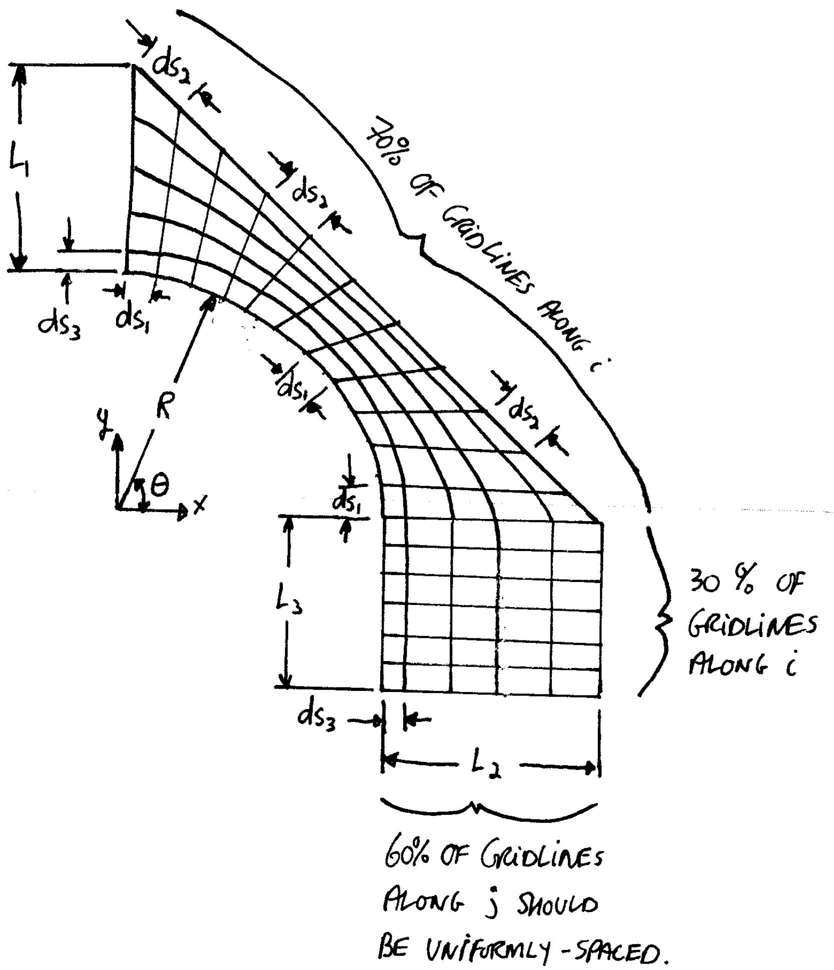
(a) Find $\partial F_1 / \partial U_1$

(b) Find $\partial F_3 / \partial U_4$

Hint: $\rho H = \rho E + P$.

Question #4

Consider the following grid:



with $R = 1$ m, $L_1 = 1.5$ m, $L_2 = 1.2$ m, $L_3 = 0.5$ m, and $ds_3 = 10^{-4}$ m. Note that ds_1 and ds_2 are uniform in the range $0^\circ \leq \theta \leq 90^\circ$ (but $ds_1 \neq ds_2$) and that the mesh should have 123 grid lines along i and 56 grid lines along j. Do the following:

(a) Outline the gridding strategy

(b) Write the code that would generate this grid. The grid should be such that there is no sudden change in mesh spacing at any location.

Answers

3. (a) $u\frac{\rho_2}{\rho}$, (b) $\frac{\rho_1 R_1 + \rho_2 R_2}{c_{v1}\rho_1 + c_{v2}\rho_2}$