Computational Aerodynamics Assignment 9 — Numerical Error

Question #1

For your design project case and the baseline mesh (mf=1), estimate the solution convergence error as a function of the maximum residual or as a function of tmax (whichever is better suited). The solution convergence error is to be estimated by monitoring either the drag coefficient or the average stagnation pressure. For this purpose, make a table as follows:

$(R_\Delta)_{\max}$ (or $t_{\max}$)	C_D (or $P_\circ$)	Estimate of relative error on C_D (or $P_\circ$)
...	...	...
...	...	...
...	...	...
...	...	...

Choose enough values of $(R_\Delta)_{\max}$ (or of $t_{\max}$) so that the trend becomes apparent. Explain clearly how all the values in the table are obtained (show the calculations for each number).

Question #2

Using an optimal value of $(R_\Delta)_{\max}$ (or $t_{\max}$) as determined in Question #1 above, do a grid convergence study to assess the grid-induced error for your design problem case. The grid convergence study should be done with a grid refinement ratio $r = 1.3$ and with at least 4 different grid levels (or more if the solution is not within the asymptotic range of convergence). As much as possible and as accurately as possible for each grid level, estimate:

(a) the effective order of accuracy p

(b) C_D (or $P_\circ$)

(c) the grid convergence index (GCI) for C_D (or $P_\circ$)

(d) an estimate of the exact solution of C_D (or $P_\circ$)

Make sure that your $(R_\Delta)_{\max}$ (or $t_{\max}$) yields an error significantly less than the one you get with the GCI. If not, rerun your cases with a more stringent value of $(R_\Delta)_{\max}$ (or $t_{\max}$). Present your results in tabular form and explain clearly how each entry was obtained (show the calculations and formulas used). Also determine, if possible, at which mesh level the solution is within the asymptotic range of convergence.

Question #3

Starting from Taylor series expansion of a first derivative, show that the following holds:

$$\epsilon_{\mathrm{f}}^{\mathrm{disc}} = \left((\delta_x \phi)_{\mathrm{f}} - (\delta_x \phi)_{\mathrm{c}} \right) \Big/ \left(1 - \left(\frac{\Delta x_{\mathrm{c}}}{\Delta x_{\mathrm{f}}} \right)^p \right)$$

with

$$\epsilon_{\mathrm{f}}^{\mathrm{disc}} \equiv (\delta_x \phi)_{\mathrm{f}} - \partial_x \phi$$

Outline all assumptions and limitations if any.

Due on Tuesday June 18th at 16:30. Do all problems.