Computational Aerodynamics
Assignment 6 — Flux Discretization I

Question #1

Starting from the scalar advection equation:

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = 0$$

and assuming a negative wave speed:

$$a < 0$$

show that when limiting the second order terms such that they adhere to the rule of the positive coefficients, a second-order upwinded slope-limited scheme can be obtained as:

$$u_{i+1/2} = u_{i+1} + \phi_{i+1/2}\frac{1}{2}(u_{i+1} - u_{i+2})$$

with the limiter function:

$$0 \leq \phi_{i+1/2} \leq 2r_i$$

and the ratio of successive gradients:

$$r_i = \frac{u_i - u_{i+1}}{u_{i+1} - u_{i+2}}$$

Question #2

Derive a 5th order accurate WENO discretization of $u_{i+1/2}$ for the advection equation with $a > 0$. Specifically, do the following:

(a) Find the three third-order accurate 2nd degree polynomials $P_1(x)$, $P_2(x)$, $P_3(x)$ obtained from u_{i-2}, u_{i-1}, u_i, u_{i+1}, and u_{i+2}.

(b) Using $P_1(x)$, $P_2(x)$, $P_3(x)$ found in (a), evaluate $u^{(1)}_{i+\frac{1}{2}}$, $u^{(2)}_{i+\frac{1}{2}}$, $u^{(3)}_{i+\frac{1}{2}}$.

(c) Find a fifth-order accurate 4th degree polynomial $P_4(x)$ that goes through the nodes mentioned in (a).

(d) Determine the optimal weights γ_1, γ_2, γ_3.

(e) Determine the coefficient of smoothness β_1 associated with $P_1(x)$.

(f) Express $u_{i+1/2}$ as a function of β_j and $u^{(j)}_{i+1/2}$ with $j = 1, 2, 3$.

Answers

2. $u^{(1)}_{i+\frac{1}{2}} = \frac{3}{8}u_{i-2} - \frac{5}{4}u_{i-1} + \frac{15}{8}u_i$, $u^{(2)}_{i+\frac{1}{2}} = -\frac{1}{8}u_{i-1} + \frac{3}{4}u_i + \frac{3}{8}u_{i+1}$,
$u^{(3)}_{i+\frac{1}{2}} = \frac{3}{8}u_i + \frac{3}{4}u_{i+1} - \frac{1}{8}u_{i+2}$, $\gamma_1 = \frac{1}{16}$, $\gamma_2 = \frac{5}{8}$, $\gamma_3 = \frac{5}{16}$,
$\beta_1 = \frac{1}{3}\left(4u^2_{i-2} - 19u_{i-2}u_{i-1} + 25u^2_{i-1} + 11u_{i-2}u_i - 31u_{i-1}u_i + 10u_i^2\right)$

Due on Thursday May 23rd at 16:30. Do both questions.