

# Computational Aerodynamics

## Assignment 6 — Flux Discretization I

Due on Thursday May 23rd at 16:30. Do both questions.

### Question #1

Starting from the scalar advection equation:

$$\frac{\partial u}{\partial t} + a \frac{\partial u}{\partial x} = 0$$

and assuming a negative wave speed:

$$a < 0$$

show that when limiting the second order terms such that they adhere to the rule of the positive coefficients, a second-order upwinded slope-limited scheme can be obtained as:

$$u_{i+1/2} = u_{i+1} + \phi_{i+1/2} \frac{1}{2} (u_{i+1} - u_{i+2})$$

with the limiter function:

$$0 \leq \phi_{i+1/2} \leq 2r_i$$

and the ratio of successive gradients:

$$r_i = \frac{u_i - u_{i+1}}{u_{i+1} - u_{i+2}}$$

### Question #2

Derive a 5th order accurate WENO discretization of  $u_{i+1/2}$  for the advection equation with  $a > 0$ . Specifically, do the following:

- Find the three third-order accurate 2nd degree polynomials  $P_1(x)$ ,  $P_2(x)$ ,  $P_3(x)$  obtained from  $u_{i-2}$ ,  $u_{i-1}$ ,  $u_i$ ,  $u_{i+1}$ , and  $u_{i+2}$ .
- Using  $P_1(x)$ ,  $P_2(x)$ ,  $P_3(x)$  found in (a), evaluate  $u_{i+\frac{1}{2}}^{(1)}$ ,  $u_{i+\frac{1}{2}}^{(2)}$ ,  $u_{i+\frac{1}{2}}^{(3)}$ .
- Find a fifth-order accurate 4th degree polynomial  $P_4(x)$  that goes through the nodes mentioned in (a).
- Determine the optimal weights  $\gamma_1$ ,  $\gamma_2$ ,  $\gamma_3$ .
- Determine the coefficient of smoothness  $\beta_1$  associated with  $P_1(x)$ .
- Express  $u_{i+1/2}$  as a function of  $\beta_j$  and  $u_{i+1/2}^{(j)}$  with  $j = 1, 2, 3$ .

### Answers

$$2. \quad u_{i+\frac{1}{2}}^{(1)} = \frac{3}{8}u_{i-2} - \frac{5}{4}u_{i-1} + \frac{15}{8}u_i, \quad u_{i+\frac{1}{2}}^{(2)} = -\frac{1}{8}u_{i-1} + \frac{3}{4}u_i + \frac{3}{8}u_{i+1},$$

$$u_{i+\frac{1}{2}}^{(3)} = \frac{3}{8}u_i + \frac{3}{4}u_{i+1} - \frac{1}{8}u_{i+2}, \quad \gamma_1 = \frac{1}{16}, \quad \gamma_2 = \frac{5}{8}, \quad \gamma_3 = \frac{5}{16},$$

$$\beta_1 = \frac{1}{3} (4u_{i-2}^2 - 19u_{i-2}u_{i-1} + 25u_{i-1}^2 + 11u_{i-2}u_i - 31u_{i-1}u_i + 10u_i^2)$$