2017 Computational Aerodynamics Final Exam

When is the best time for you?

Thu 8th Jun 16:30--19:30	5
Fri 9th Jun 15:30--18:30	4
Tue 13th Jun 16:30--19:30	3
Thu 15th Jun 16:30--19:30	2
Fri 16th Jun 15:30--18:30	1
Tue 20th Jun 16:30--19:30	1

Poll ended at 1:52 am on Tuesday May 30th 2017. Total votes: 16. Total voters: 8.

Please select your favourite time slots to have the final exam. We will decide the time and date in class next week based on your votes.

The final exam will take place on Thursday June 8th from 16:30 — 19:30. The material includes Assignments #2 to #8. 3 out of 6 questions will be taken from the Assignments, and 3 will be new but similar to those in the Assignments.

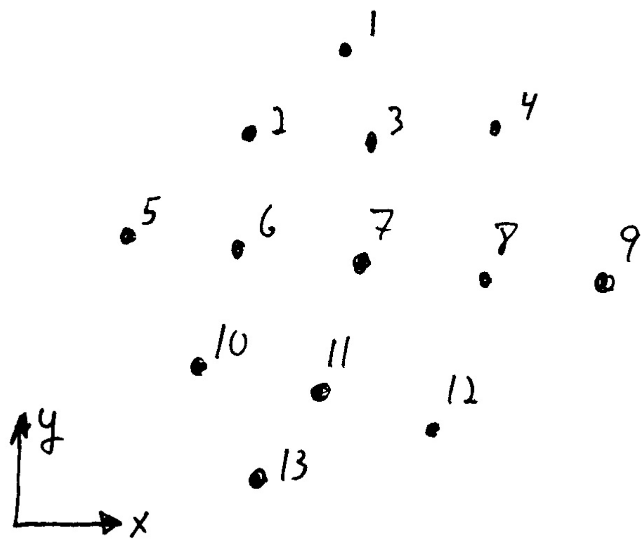
The final exam will take place in room 9302.

Thursday June 8th 2017
16:30 — 19:30

NO NOTES OR BOOKS; USE INTRODUCTION TO CFD TABLES THAT WERE DISTRIBUTED; ALL QUESTIONS HAVE EQUAL VALUE; ANSWER ALL 6 QUESTIONS.

Question #1

Consider the following nodes in the x-y plane:



with the following associated properties:

Node	x, mm	y, mm	ρ, kg/m^3
1	530	-90	1.0
2	400	-210	1.05
3	570	-220	1.05
4	750	-200	1.1
5	220	-360	1.05
6	380	-380	1.1
7	550	-400	1.15
8	730	-410	1.2
9	900	-420	1.25
10	320	-540	1.15
11	500	-580	1.20
12	650	-630	1.25
13	410	-700	1.30

For the nodes shown above, do the following:

(a) Find Ω at node 7 using second-order accurate stencils for the metrics.

(b) Find the cell area at node 7 using a method of your choice and compare it with Ω found in (a).

Question #2

Starting from the 1st law of thermo

$\mathrm{d}(mh) - V\mathrm{d}P = \delta Q - \delta W$

the y momentum equation in 1D

$$\rho\frac{\partial v}{\partial t} + \rho v\frac{\partial v}{\partial y} = -\frac{\partial P}{\partial y}$$

show that the total energy transport equation for a fluid corresponds to:

$$\frac{\partial \rho E}{\partial t} + \frac{\partial \rho v H}{\partial y} = 0$$

with the total energy $E \equiv e + \frac{1}{2}q^2$, the total enthalpy $H \equiv h + \frac{1}{2}q^2$, q the speed of the flow, and T the temperature.

Question #3

Starting from Taylor series expansion of a first derivative, show that the following holds:

$$\epsilon_{\rm f}^{\rm disc} = \left((\delta_x\phi)_{\rm f} - (\delta_x\phi)_{\rm c}\right) \Big/ \left(1 - \left(\frac{\Delta x_{\rm c}}{\Delta x_{\rm f}}\right)^p\right)$$

with

$$\epsilon_{\rm f}^{\rm disc} \equiv (\delta_x\phi)_{\rm f} - \partial_x\phi$$

Outline all assumptions and limitations if any.

Question #4

Consider a system of equations $\partial U/\partial t + \partial F/\partial x = 0$ with $F = AU$, $A = L^{-1}\Lambda L$ and with:

$$\Lambda = \begin{bmatrix} u & 0 \\ 0 & u-a \end{bmatrix} \quad L = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \quad U = \begin{bmatrix} u \\ a \end{bmatrix} \quad F = \begin{bmatrix} u^2 + 2a^2 \\ a(u-a) \end{bmatrix}$$

The node properties correspond to:

Node	u	a
$i-1$	0	100
i	0	110
$i+1$	0	105
$i+2$	0	100

For the primitive variable vector set to:

$$Z = U = \begin{bmatrix} u \\ a \end{bmatrix}$$

and using a second-order-upwind slope-limited FDS scheme with reconstruction evolution and arithmetic averaging, do the following:

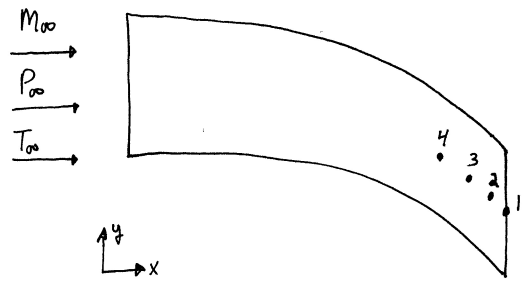
(a) Find the primitive variable vector on the left and right sides of the interface, $Z_{\rm L}$ and $Z_{\rm R}$.

(b) Find the flux at the interface $F_{i+1/2}$.

Note: both u and a are non-dimensional.

Question #5

Consider a computational domain as follows:



with the gas constant $R = 286$ J/kgK, the ratio of specific heats $\gamma = 1.4$, $P_\infty = 2$ atm, $T_\infty = 300$ K, and $M_\infty = 0.8$. Knowing the properties at nodes 1, 2, 3, and 4 at time level n:

Node	x, m	y, m	P^n, Pa	T^n, K	u^n, m/s	v^n, m/s
1	1.00	1.00	90000	310	400	30
2	0.99293	1.00707	90000	310	380	20
3	0.97879	1.02121	90000	350	350	10
4	0.96464	1.03535	90000	330	340	0

Do the following:

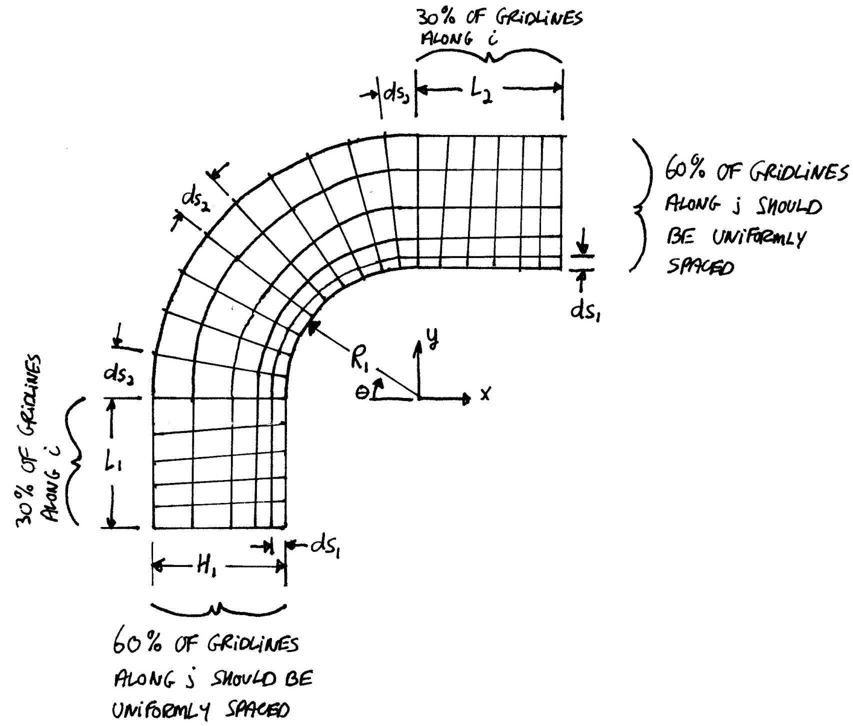
(a) Determine what kind of boundary condition node 1 is.

(b) Find the pressure and temperature at time level $n+1$ at node 1 using a 2nd degree polynomial to extrapolate the properties from the inner nodes 2, 3 and 4.

Hint: the spacing between the nodes 4, 3, 2, and 1 can not be assumed constant.

Question #6

Consider the following grid schematic:



In the latter, $R_1 = 0.5$ m, $ds_1 = H_1/100$, $H_1 = 1.0$ m, $L_1 = 0.5$ m, and $L_2 = 0.8$ m. Note that ds_2 is uniform in the range $0^\circ \leq \theta \leq 90^\circ$ and that the mesh should have 123 grid lines along i and 56 grid lines along j. Do the following:

(a) Outline the gridding strategy

(b) Write the GRIDG code that would generate this grid. The grid should be such that there is no sudden change in mesh spacing at any location.