2018 Computational Aerodynamics Final Exam

Thursday June 14th 2018
16:30 — 19:30

NO NOTES OR BOOKS; USE INTRODUCTION TO CFD TABLES THAT WERE DISTRIBUTED; ALL QUESTIONS HAVE EQUAL VALUE; ANSWER ALL 6 QUESTIONS.

Question #1

Starting from the Euler equations

$$\frac{\partial U}{\partial t} + \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} = 0$$

and the metrics η_x , ξ_y , Ω, etc derived above in Question #1, show that the Euler equations can be written in generalized coordinates in strong conservative form as follows:

$$\frac{\partial Q}{\partial \tau} + \frac{\partial G_\xi}{\partial \xi} + \frac{\partial G_\eta}{\partial \eta} = 0$$

with

$Q \equiv \Omega \Gamma U$

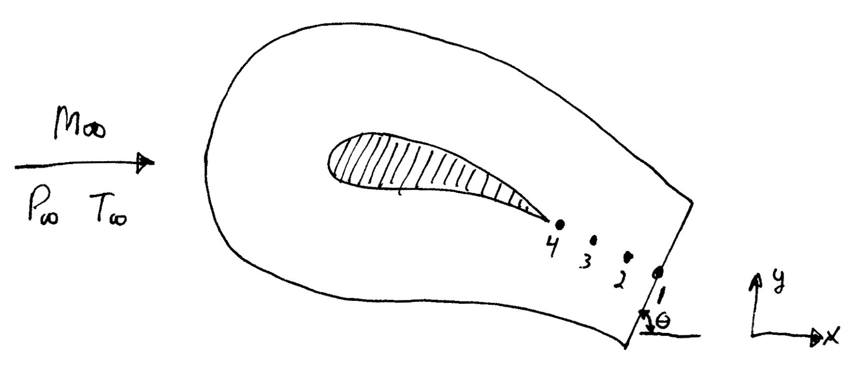
$G_\xi \equiv \Omega(\xi_x F_x + \xi_y F_y)$

$G_\eta \equiv \Omega(\eta_x F_x + \eta_y F_y)$

Outline clearly your assumptions.

Question #2

Consider the following domain:



The freestream properties correspond to $M_\infty = 0.7$, $T_\infty = 300$ K, $P_\infty = 1$ atm, $\theta = 70^\circ$ and the properties on the nodes at the iteration n are as follows:

Node	P^n, atm	T^n, K	u^n, m/s	v^n, m/s
1	1.0	300	100	-80
2	1.1	330	130	-100
3	1.1	350	150	-130
4	1.2	380	170	-160

Knowing that the node spacing is constant, do the following:

(a) Find the properties at iteration $n+1$ at node 1 using a 1st degree polynomial to extrapolate the properties.

Outline clearly what kind of boundary condition (subsonic inflow/outflow, supersonic inflow/outflow) you are choosing and why.

Question #3

Starting from Taylor series expansion of a first derivative, show that the following holds:

$$\epsilon_{\rm f}^{\rm disc} = \Big((\delta_x \phi)_{\rm f} - (\delta_x \phi)_{\rm c} \Big) \Big/ \left(1 - \left(\frac{\Delta x_{\rm c}}{\Delta x_{\rm f}} \right)^p \right)$$

with

$$\epsilon_{\rm f}^{\rm disc} \equiv (\delta_x \phi)_{\rm f} - \partial_x \phi$$

Outline all assumptions and limitations if any.

Question #4

Consider the following system of equations:

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} = 0$$

with

$$U = \begin{bmatrix} \rho_1 \\ \rho_2 \\ \rho u \\ \rho E \end{bmatrix} = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} \quad \text{and} \quad F = \begin{bmatrix} \rho_1 u \\ \rho_2 u \\ \rho u^2 + P \\ \rho u H \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}$$

with

$$E = \frac{\rho_1}{\rho} e_1 + \frac{\rho_2}{\rho} e_2 + \frac{u^2}{2}$$

$$P = (\rho_1 R_1 + \rho_2 R_2) T$$

$$\rho = \rho_1 + \rho_2$$

$$H = E + \frac{P}{\rho}$$

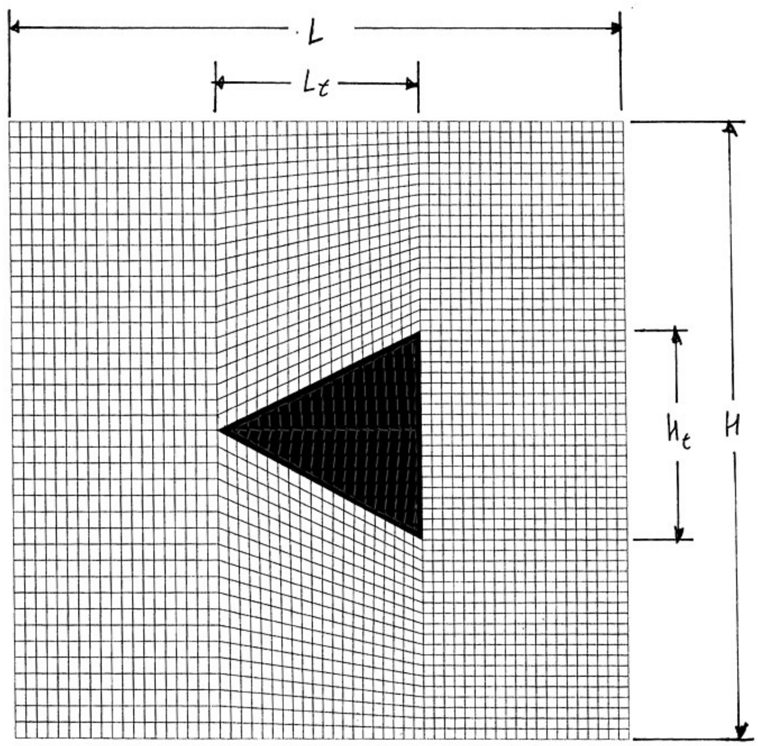
$$e_1 = \xi_1 + \xi_2 T + \xi_3 T^2 + \xi_4 T^3 + \xi_5 T^4$$

$$e_2 = \xi_6 T$$

and with ξ_1, ξ_2, ξ_3, ξ_4, ξ_5, ξ_6, R_1, R_2 some constants. Find $\partial F_3 / \partial U_4$ within the flux Jacobian.

Question #5

You wish to simulate an external flow interacting with a triangle as follows:



The flow infinitely far from the triangle has a temperature of 300 K, a pressure of 20 kPa, and a velocity (in the right direction) of 600 m/s. Knowing that the grid is made of 60 gridlines in each dimension, and that $L = 2.1$ m, $H = 2.1$ m, $L_\mathrm{t} = 0.7$ m, $H_\mathrm{t} = 0.7$ m, that the grid on each side of the domain is uniformly spaced, and that the triangle is centered within the domain, write the code needed within the Grid() and the Bdry() modules to simulate this problem. Note: you can find a control file template within the last 2 sheets of the tables.

Question #6

You wish to solve numerically the following scalar equation:

$$\frac{\partial u}{\partial t} + \frac{\partial f}{\partial x} = 0$$

with $f = \frac{1}{2}u^2$. At a certain time level, u corresponds to:

Node	x	u

1	0.0	4
2	0.1	3
3	0.2	3
4	0.3	4
5	0.4	5
6	0.5	8
7	0.6	11
8	0.7	12

Using a WENO 2nd-3rd order interpolation of the primitive u reconstructed over a FDS scheme with arithmetic averaging, it is desired to find the flux at the interface between node 4 and node 5, i.e. $f_{4.5}$. For this purpose, do the following:

(a) Find u_{L} between node 4 and 5 using WENO3.

(b) Find u_{R} between node 4 and 5 using WENO3.

(c) Find $f_{4.5}$ using FDS with u_{L} and u_{R} found in (a) and (b).