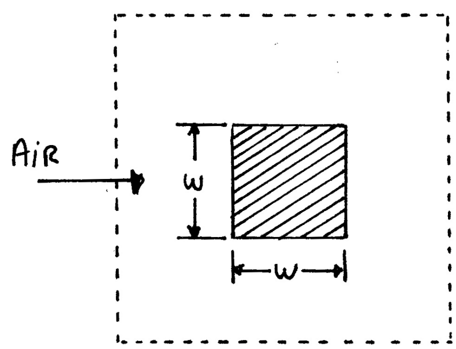
2019 Computational Aerodynamics Final Exam

Tuesday June 18th 2019
16:30 — 19:30

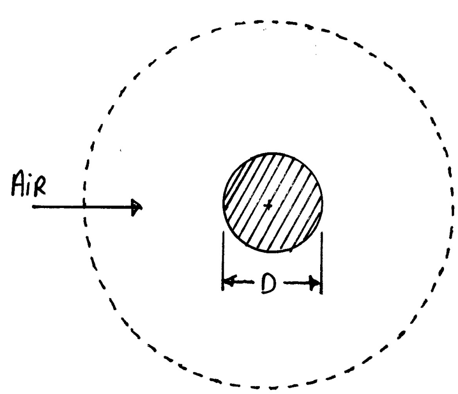
NO NOTES OR BOOKS; USE COMPUTATIONAL AERODYNAMICS TABLES THAT WERE DISTRIBUTED; ALL QUESTIONS HAVE EQUAL VALUE; ANSWER ALL 6 QUESTIONS.

Question #1

Consider air interacting with a long rod of square cross-section as follows:



or a infinitely long cylinder as follows:



The air in the freestream has a Mach number of 2.5, a pressure of 5 kPa, and a temperature of 300 K. The diameter of the cylinder is of $D = 0.1$ m. The width of the cross section is of $W = 0.1$ m. The goal is to determine the drag coefficient C_D as accurately as possible while using as few computational resources as possible. Design a mesh that is optimal for this problem (i.e., that will yield the smallest product of error and computational time). Set the viscosity and thermal conductivity to zero. Specifically, for either the long cylinder or the square long rod problems above, do the following:

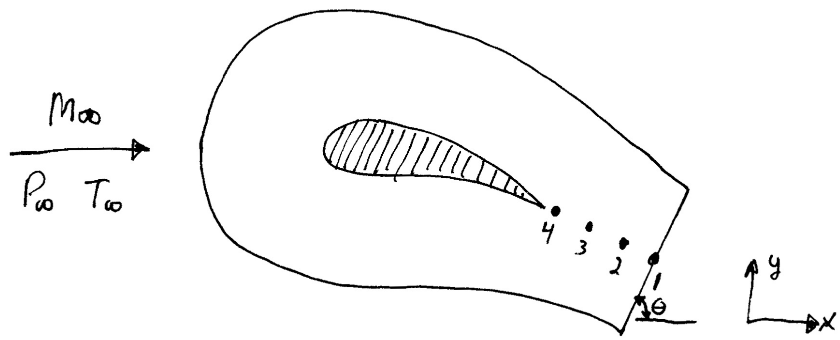
(a) In the Grid() module, create a mesh function of the mesh factor "mf". Thus, mf = 1 will give the coarsest mesh, mf = 2 will give the same mesh as mf = 1 but with every cell divided in 4 cells, mf = 4 will give the same mesh as mf = 2 but with every cell divided in 4 cells, etc.

(b) In the Bdry() module set up the boundary conditions correctly.

(c) In the Init() module set up the initial conditions correctly.

Question #2

Consider the following domain:



The freestream properties correspond to $M_\infty = 0.7$, $T_\infty = 300$ K, $P_\infty = 1$ atm, $\theta = 70°$ and the properties on the nodes at the iteration n are as follows:

Node	P^n, atm	T^n, K	u^n, m/s	v^n, m/s
1	1.0	300	100	-80
2	1.1	330	130	-100
3	1.1	350	150	-130
4	1.2	380	170	-160

Knowing that the node spacing is constant, do the following:

(a) Find the properties at iteration $n + 1$ at node 1 using a 1st degree polynomial to extrapolate the properties.

Outline clearly what kind of boundary condition (subsonic inflow/outflow, supersonic inflow/outflow) you are choosing and why.

Question #3

Consider the following system of equations:

$$\frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} = 0$$

with

$$U = \begin{bmatrix} \rho_1 \\ \rho_2 \\ \rho u \\ \rho E \end{bmatrix} = \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} \quad \text{and} \quad F = \begin{bmatrix} \rho_1 u \\ \rho_2 u \\ \rho u^2 + P \\ \rho u H \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}$$

with

$$E = \frac{\rho_1}{\rho} e_1 + \frac{\rho_2}{\rho} e_2 + \frac{u^2}{2}$$

$$P = (\rho_1 R_1 + \rho_2 R_2)\, T$$

$$\rho = \rho_1 + \rho_2$$

$$H = E + \frac{P}{\rho}$$

$$e_1 = \xi_1 + \xi_2 T + \xi_3 T^2 + \xi_4 T^3 + \xi_5 T^4$$

$$e_2 = \xi_6 T$$

and with ξ_1, ξ_2, ξ_3, ξ_4, ξ_5, ξ_6, R_1, R_2 some constants. Find $\partial F_3 / \partial U_4$ within the flux Jacobian.

Question #4

Consider the 2D advection equation:

$$\frac{\partial u}{\partial t} + a \frac{\partial u}{\partial x} + b \frac{\partial u}{\partial y} = 0$$

with a and b the wave speeds along x and y respectively. Derive appropriate first-order upwinded stencils for both spatial derivatives. The mesh has uniform spacing Δx along x and uniform spacing Δy along y. The stencils must be valid whether a and b are positive or negative. Consider the case of a and b being constants. Using the principle of the positive coefficients, find the maximum time step that ensures that the solution has no spurrious oscillations.

Question #5

You are simulating the Euler equations in 1D for a gas with $R = 286$ J/kgK, $c_p = 1000$ J/kgK and $\gamma = 1.4$. At one time step, the node properties are as follows:

Node	u, m/s	ρ, kg/m^3	T, K
1	13	1	300
2	12	1	306
3	10	1	310
4	11	1.2	343
5	12	1.3	340
6	12	1.4	335

7	13	1.5	346

You wish to find the flux at the interface between node 3 and node 4 using a FDS scheme with Roe averaging turned second-order accurate through the reconstruction-evolution (MUSCL) strategy. Specifically, do the following:

(a) Find ρ, u, and T on the left and right of the interface using 1st degree polynomials (without limiting).

(b) Using Roe averaging, find the properties $\rho_{3.5}$, $u_{3.5}$, and $T_{3.5}$ that are needed to build the matrix $|A|$ at the interface between node 3 and 4.

Question #6

You measure a certain property ϕ at a certain location in the computational domain as a function of the mesh size as follows:

Case	Mesh Size	ϕ
1.	125×125	10.000
2.	250×250	5.000
3.	500×500	2.000
4.	707×707	1.380
5.	1000×1000	1.000
6.	2000×2000	0.8010
7.	2828×2828	0.7726
8.	4000×4000	0.7600
9.	8000×8000	0.7520
10.	16000×16000	0.7504

Knowing that the mesh is uniformly spaced for all cases, do the following:

(a) Determine when the solution is within the asymptotic range of convergence. Identify clearly at the coarsest mesh that is within the asymptotic range.

(b) Determine the GCI on the coarsest mesh that is within the asymptotic range.

(c) Estimate the exact solution to ϕ using the GCI obtained in (b).