

Computational Aerodynamics

Assignment 7 — Flux Discretization II

Question #1

Consider a system of equations $\partial U / \partial t + \partial F / \partial x = 0$ with $F = AU$, $A = L^{-1}\Lambda L$ and with:

$$\Lambda = \begin{bmatrix} u & 0 \\ 0 & u - a \end{bmatrix} \quad L = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \quad U = \begin{bmatrix} u \\ a \end{bmatrix}$$

The node properties correspond to:

Node	u , m/s	a , m/s
$i - 1$	0	100
i	10	110
$i + 1$	9	105
$i + 2$	-10	100

Do the following:

- Find $F_{i+1/2}^+$ with a minmod2 limiter 2nd-order FVS scheme.
- Find $F_{i+1/2}^-$ with a minmod2 limiter 2nd-order FVS scheme.
- Find $F_{i+1/2}$ with a minmod2 limiter 2nd-order FVS scheme.

Question #2

Consider a system of equations $\partial U / \partial t + \partial F / \partial x = 0$ with $F = AU$, $A = L^{-1}\Lambda L$ and with:

$$\Lambda = \begin{bmatrix} u & 0 \\ 0 & u - a \end{bmatrix} \quad L = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \quad U = \begin{bmatrix} u \\ a \end{bmatrix} \quad F = \begin{bmatrix} u^2 + 2a^2 \\ a(u - a) \end{bmatrix}$$

The node properties correspond to:

Node	u	a
$i - 1$	0	100
i	0	110
$i + 1$	0	105
$i + 2$	0	100

For the primitive variable vector set to:

$$Z = U = \begin{bmatrix} u \\ a \end{bmatrix}$$

and using a second-order-upwind slope-limited FDS scheme with the minmod2 limiter, reconstruction evolution, and arithmetic averaging, do the following:

- Find the primitive variable vector on the left and right sides of the interface, Z_L and Z_R .
- Find the flux at the interface $F_{i+1/2}$.

Note: both u and a are non-dimensional.

Question #3

You wish to solve numerically the following scalar equation:

$$\frac{\partial u}{\partial t} + \frac{\partial f}{\partial x} = 0$$

with $f = \frac{1}{2}u^2$. At a certain time level, u corresponds to:

Node	x	u
1	0.0	4
2	0.1	3
3	0.2	3
4	0.3	4
5	0.4	5
6	0.5	8
7	0.6	11
8	0.7	12

Using a WENO 2nd-3rd order interpolation of the primitive u reconstructed over a FDS scheme with arithmetic averaging and with optimal weights set to $\gamma_0 = \frac{1}{3}$ and $\gamma_1 = \frac{2}{3}$, it is desired to find the flux at the interface between node 4 and node 5, i.e. $f_{4.5}$. For this purpose, do the following:

- Find u_L between node 4 and 5 using WENO3.
- Find u_R between node 4 and 5 using WENO3.
- Find $f_{4.5}$ using FDS with u_L and u_R found in (a) and (b).

Answers

- 2300, 0 m²/s²; 20190, -10080 m²/s².
- 23928.125, -11692.1875.
- 4.5, $\frac{1465}{326}$, $10\frac{26569}{212552}$.

Due on May 30th at 16:30. Do Questions #2 and #3 only.