Computational Aerodynamics
Assignment 7 — Flux Discretization II

Question #1

Consider a system of equations $\partial U/\partial t + \partial F/\partial x = 0$ with $F = AU$, $A = L^{-1}\Lambda L$ and with:

$$\Lambda = \begin{bmatrix} u & 0 \\ 0 & u-a \end{bmatrix} \quad L = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \quad U = \begin{bmatrix} u \\ a \end{bmatrix}$$

The node properties correspond to:

Node	u, m/s	a, m/s
$i-1$	0	100
i	10	110
$i+1$	9	105
$i+2$	-10	100

Do the following:

(a) Find $F^{+}_{i+1/2}$ with a minmod2 limiter 2nd-order FVS scheme.

(b) Find $F^{-}_{i+1/2}$ with a minmod2 limiter 2nd-order FVS scheme.

(c) Find $F_{i+1/2}$ with a minmod2 limiter 2nd-order FVS scheme.

Question #2

Consider a system of equations $\partial U/\partial t + \partial F/\partial x = 0$ with $F = AU$, $A = L^{-1}\Lambda L$ and with:

$$\Lambda = \begin{bmatrix} u & 0 \\ 0 & u-a \end{bmatrix} \quad L = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \quad U = \begin{bmatrix} u \\ a \end{bmatrix} \quad F = \begin{bmatrix} u^2 + 2a^2 \\ a(u-a) \end{bmatrix}$$

The node properties correspond to:

Node	u	a
$i-1$	0	100
i	0	110
$i+1$	0	105
$i+2$	0	100

For the primitive variable vector set to:

$$Z = U = \begin{bmatrix} u \\ a \end{bmatrix}$$

and using a second-order-upwind slope-limited FDS scheme with the minmod2 limiter, reconstruction evolution, and arithmetic averaging, do the following:

(a) Find the primitive variable vector on the left and right sides of the interface, Z_{L} and Z_{R}.

(b) Find the flux at the interface $F_{i+1/2}$.

Note: both u and a are non-dimensional.

Question #3

You wish to solve numerically the following scalar equation:

$$\frac{\partial u}{\partial t} + \frac{\partial f}{\partial x} = 0$$

with $f = \frac{1}{2}u^2$. At a certain time level, u corresponds to:

Node	x	u
1	0.0	4
2	0.1	3
3	0.2	3
4	0.3	4
5	0.4	5
6	0.5	8
7	0.6	11
8	0.7	12

Using a WENO 2nd-3rd order interpolation of the primitive u reconstructed over a FDS scheme with arithmetic averaging and with optimal weights set to $\gamma_0 = \frac{1}{3}$ and $\gamma_1 = \frac{2}{3}$, it is desired to find the flux at the interface between node 4 and node 5, i.e. $f_{4.5}$. For this purpose, do the following:

(a) Find u_{L} between node 4 and 5 using WENO3.

(b) Find u_{R} between node 4 and 5 using WENO3.

(c) Find $f_{4.5}$ using FDS with u_{L} and u_{R} found in (a) and (b).

Answers

1. 2300, 0 $\mathrm{m^2/s^2}$; 20190, -10080 $\mathrm{m^2/s^2}$.
2. 23928.125, -11692.1875.
3. 4.5, $\frac{1465}{326}$, $10\frac{26569}{212552}$.

Due on May 30th at 16:30. Do Questions #2 and #3 only.