

# Computational Aerodynamics

## Assignment 7 — Flux Discretization II

### Question #1

Consider a system of equations  $\partial U/\partial t + \partial F/\partial x = 0$  with  $F = AU$ ,  $A = L^{-1}\Lambda L$  and with:

$$\Lambda = \begin{bmatrix} u & 0 \\ 0 & u-a \end{bmatrix} \quad L = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \quad U = \begin{bmatrix} u \\ a \end{bmatrix}$$

The node properties correspond to:

Node	$u$ , m/s	$a$ , m/s
$i-1$	0	100
$i$	10	110
$i+1$	9	105
$i+2$	-10	100

Do the following:

- Find  $F_{i+1/2}^+$  with a minmod2 limiter 2nd-order FVS scheme.
- Find  $F_{i+1/2}^-$  with a minmod2 limiter 2nd-order FVS scheme.
- Find  $F_{i+1/2}$  with a minmod2 limiter 2nd-order FVS scheme.

### Question #2

Consider a system of equations  $\partial U/\partial t + \partial F/\partial x = 0$  with  $F = AU$ ,  $A = L^{-1}\Lambda L$  and with:

$$\Lambda = \begin{bmatrix} u & 0 \\ 0 & u-a \end{bmatrix} \quad L = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \quad U = \begin{bmatrix} u \\ a \end{bmatrix} \quad F = \begin{bmatrix} u^2 + 2a^2 \\ a(u-a) \end{bmatrix}$$

The node properties correspond to:

Node	$u$	$a$
$i-1$	0	100
$i$	0	110
$i+1$	0	105
$i+2$	0	100

For the primitive variable vector set to:

$$Z = U = \begin{bmatrix} u \\ a \end{bmatrix}$$

and using a second-order-upwind slope-limited FDS scheme with the minmod2 limiter, reconstruction evolution, and arithmetic averaging, do the following:

(a) Find the primitive variable vector on the left and right sides of the interface,  $Z_L$  and  $Z_R$ .

(b) Find the flux at the interface  $F_{i+1/2}$ .

Note: both  $u$  and  $a$  are non-dimensional.

### Question #3

You wish to solve numerically the following scalar equation:

$$\frac{\partial u}{\partial t} + \frac{\partial f}{\partial x} = 0$$

with  $f = \frac{1}{2}u^2$ . At a certain time level,  $u$  corresponds to:

Node	$x$	$u$
1	0.0	4
2	0.1	3
3	0.2	3
4	0.3	4
5	0.4	5
6	0.5	8
7	0.6	11
8	0.7	12

Using a WENO 2nd-3rd order interpolation of the primitive  $u$  reconstructed over a FDS scheme with arithmetic averaging and with optimal weights set to  $\gamma_0 = \frac{1}{3}$  and  $\gamma_1 = \frac{2}{3}$ , it is desired to find the flux at the interface between node 4 and node 5, i.e.  $f_{4.5}$ . For this purpose, do the following:

- Find  $u_L$  between node 4 and 5 using WENO3.
- Find  $u_R$  between node 4 and 5 using WENO3.
- Find  $f_{4.5}$  using FDS with  $u_L$  and  $u_R$  found in (a) and (b).

### Answers

- 2300, 0 m<sup>2</sup>/s<sup>2</sup>; 20190, -10080 m<sup>2</sup>/s<sup>2</sup>.
- 23928.125, -11692.1875.
- 4.5,  $\frac{1465}{326}$ ,  $10\frac{26569}{212552}$ .

**Due on May 30th at 16:30. Do Questions #2 and #3 only.**