# Computational Aerodynamics
# Assignment 7 — Flux Discretization II

### Question #1

Consider a system of equations $\partial U/\partial t + \partial F/\partial x = 0$ with $F = AU$, $A = L^{-1}\Lambda L$ and with:

$$\Lambda = \begin{bmatrix} u & 0 \\ 0 & u-a \end{bmatrix} \quad L = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \quad U = \begin{bmatrix} u \\ a \end{bmatrix}$$

The node properties correspond to:

| Node | $u$, m/s | $a$, m/s |
|---|---|---|
| $i-1$ | 0 | 100 |
| $i$ | 10 | 110 |
| $i+1$ | 9 | 105 |
| $i+2$ | -10 | 100 |

Do the following:

(a) Find $F^{+}_{i+1/2}$ with a minmod2 limiter 2nd-order FVS scheme.

(b) Find $F^{-}_{i+1/2}$ with a minmod2 limiter 2nd-order FVS scheme.

(c) Find $F_{i+1/2}$ with a minmod2 limiter 2nd-order FVS scheme.

### Question #2

Consider a system of equations $\partial U/\partial t + \partial F/\partial x = 0$ with $F = AU$, $A = L^{-1}\Lambda L$ and with:

$$\Lambda = \begin{bmatrix} u & 0 \\ 0 & u-a \end{bmatrix} \quad L = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \quad U = \begin{bmatrix} u \\ a \end{bmatrix} \quad F = \begin{bmatrix} u^2 + 2a^2 \\ a(u-a) \end{bmatrix}$$

The node properties correspond to:

| Node | $u$ | $a$ |
|---|---|---|
| $i-1$ | 0 | 100 |
| $i$ | 0 | 110 |
| $i+1$ | 0 | 105 |
| $i+2$ | 0 | 100 |

For the primitive variable vector set to:

$$Z = U = \begin{bmatrix} u \\ a \end{bmatrix}$$

and using a second-order-upwind slope-limited FDS scheme with the minmod2 limiter, reconstruction evolution, and arithmetic averaging, do the following:

(a) Find the primitive variable vector on the left and right sides of the interface, $Z_{\mathrm{L}}$ and $Z_{\mathrm{R}}$.

(b) Find the flux at the interface $F_{i+1/2}$.

Note: both $u$ and $a$ are non-dimensional.

### Question #3

You wish to solve numerically the following scalar equation:

$$\frac{\partial u}{\partial t} + \frac{\partial f}{\partial x} = 0$$

with $f = \frac{1}{2}u^2$. At a certain time level, $u$ corresponds to:

| Node | $x$ | $u$ |
|---|---|---|
| 1 | 0.0 | 4 |
| 2 | 0.1 | 3 |
| 3 | 0.2 | 3 |
| 4 | 0.3 | 4 |
| 5 | 0.4 | 5 |
| 6 | 0.5 | 8 |
| 7 | 0.6 | 11 |
| 8 | 0.7 | 12 |

Using a WENO 2nd-3rd order interpolation of the primitive $u$ reconstructed over a FDS scheme with arithmetic averaging and with optimal weights set to $\gamma_0 = \frac{1}{3}$ and $\gamma_1 = \frac{2}{3}$, it is desired to find the flux at the interface between node 4 and node 5, i.e. $f_{4.5}$. For this purpose, do the following:

(a) Find $u_{\mathrm{L}}$ between node 4 and 5 using WENO3.

(b) Find $u_{\mathrm{R}}$ between node 4 and 5 using WENO3.

(c) Find $f_{4.5}$ using FDS with $u_{\mathrm{L}}$ and $u_{\mathrm{R}}$ found in (a) and (b).

### Answers

1. 2300, 0 $\mathrm{m}^2/\mathrm{s}^2$; 20190, -10080 $\mathrm{m}^2/\mathrm{s}^2$.
2. 23928.125, -11692.1875.
3. 4.5, $\frac{1465}{326}$, $10\frac{26569}{212552}$.

**Due on May 30th at 16:30. Do Questions #2 and #3 only.**