Computational Aerodynamics Questions & Answers

Question by Student 201227148

Sir, I can not get my CFD password. I entered my email address. But I can not log in.

Try again now.

Question by Student 201427116

Professor, We learned how to find wave speed from Flux Jacobian on last class. After finding all elements of Jacobian , you showed the way to extract wave speeds from Jacobian. In this process, you found 3 eigenvalues of Jacobian which are expressed by

$\phi_1, \phi_2, and \phi_3, respectively.$

And said, these eigenvalues represent each wave speeds.

But I cannot understand the relation between eigenvalues of Jacobian and wave speed. How do I confirm that these eigenvalues should be wave speed? What relationship is involved in this process?

Consider a system of equations as follows:

$$\frac{\partial U}{\partial t} + A\frac{\partial U}{\partial x} = 0$$

Recall that the eigenvalues are such that $A = L^{-1}\Lambda L$:

$$\frac{\partial U}{\partial t} + L^{-1}\Lambda L\frac{\partial U}{\partial x} = 0$$

Multiply by L:

$$L\frac{\partial U}{\partial t} + \Lambda L\frac{\partial U}{\partial x} = 0$$

Say that a vector W exists such that $L = \partial W/\partial U$. Further, because $L = L(U)$, it follows that $W = W(U)$. Thus, we can say

$$\frac{\partial W}{\partial t} = L\frac{\partial U}{\partial t} \text{ and } \frac{\partial W}{\partial x} = L\frac{\partial U}{\partial x}$$

Substitute the RHS of the latter 2 equations in the former:

$$\frac{\partial W}{\partial t} + \Lambda\frac{\partial W}{\partial x} = 0$$

Because $\Lambda = [\phi_1,\ \phi_2,\ \phi_3]^{\rm D}$ is diagonal, the latter is simply a list of advection equations each with a wave speed ϕ_1, ϕ_2, ϕ_3. Good question: 2 points bonus.

Question by Student 201327132

Dear professor. We derived Euler equations in genrealized coordinate. First we start

$$\frac{\partial U}{\partial \tau} + \xi_x \frac{\partial F_x}{\partial \xi} + \eta_x \frac{\partial F_x}{\partial \eta} + \xi_y \frac{\partial F_y}{\partial \xi} + \eta_y \frac{\partial F_y}{\partial \eta} = 0$$

Second we multiply by Ω. But I think, we don't need to multiply Ω. Because here,

$$\frac{\partial U}{\partial \tau} + \frac{\partial F_x \xi_x}{\partial \xi} - F_x \frac{\partial \xi_x}{\partial \xi} + \frac{\partial F_x \eta_x}{\partial \eta} - F_x \frac{\partial \eta_x}{\partial \eta} + \frac{\partial F_y \xi_y}{\partial \xi} - F_y \frac{\partial \xi_y}{\partial \xi} + \frac{\partial F_y \eta_y}{\partial \eta} - F_y \frac{\partial \eta_y}{\partial \eta} = 0$$

And,

$$-F_x\left(\frac{\partial \xi_x}{\partial \xi} - \frac{\partial \eta_x}{\partial \eta}\right) = -F_x\left(\frac{\partial}{\partial x}\frac{\partial \xi}{\partial \xi} - \frac{\partial}{\partial x}\frac{\partial \eta}{\partial \eta}\right) = 0$$

I think, this is more simple equation.

$$\frac{\partial U}{\partial \tau} + \frac{\partial F_x \xi_x}{\partial \xi} + \frac{\partial F_x \eta_x}{\partial \eta} + \frac{\partial F_y \xi_y}{\partial \xi} + \frac{\partial F_y \eta_y}{\partial \eta} = 0$$

I want to know why we use Ω. Thank you.

The problem with your logic is that $\partial \xi_x / \partial \xi$ is not zero because ξ_x can vary along the the ξ coordinate. Similarly, $\partial \eta_x / \partial \eta$ is also not zero because η_x can vary along η. Good question, 2 points bonus.

Question by Student 201327132

Dear professor, I have a question about WENO. We learned about what γ_0, γ_1 became W_0, W_1 when $\beta_0 \simeq \beta_1$.

But I found that γ_0 became W_0 when $\beta_0 \gg \beta_1$. $\widetilde{W}_0 = \frac{\gamma_0}{(\epsilon+\beta_0)^2}$,$\widetilde{W}_1 = \frac{\gamma_1}{(\epsilon+\beta_1)^2}$ and

$$W_0 = \frac{\widetilde{W}_0}{\widetilde{W}_0 + \widetilde{W}_1}$$

And $\gamma_0 + \gamma_1 = 1$. So

$$\widetilde{W}_0 + \widetilde{W}_1 = \frac{1}{(\epsilon+\beta_0)^2 + (\epsilon+\beta_1)^2}$$

$$W_0 = \frac{\gamma_0}{(\epsilon+\beta_0)^2} \cdot ((\epsilon+\beta_0)^2 + (\epsilon+\beta_1)^2)$$

Because of $\beta_0 \gg \beta_1$.

$$W_0 \simeq \frac{\gamma_0}{(\epsilon+\beta_0)^2} \cdot (\epsilon+\beta_0)^2$$

Finally $W_0 = \gamma_0$. *And vice versa* W_1. *Is it ok or something wrong? I think it is not satisfied only* $\beta_0 \simeq \beta_1$.

The problem in your math is here:

$$\widetilde{W}_0 + \widetilde{W}_1 = \frac{1}{(\epsilon+\beta_0)^2 + (\epsilon+\beta_1)^2}$$

This is not correct. Rather:

$$\widetilde{W}_0 + \widetilde{W}_1 = \frac{\gamma_0}{(\epsilon+\beta_0)^2} + \frac{\gamma_1}{(\epsilon+\beta_1)^2} \tag{1}$$

$$= \frac{\gamma_0(\epsilon+\beta_1)^2 + \gamma_1(\epsilon+\beta_0)^2}{(\epsilon+\beta_0)^2(\epsilon+\beta_1)^2} \tag{2}$$

Question by Student 201327132

Dear professor. In design problem 3, Should we consider about cowl length?? If so, how do we obtain cowl length?? I've been thinking for a long time. I can not find the length. Thank you.

The cowl starts where the domain ends. So, there is no need to grid the cowl and there's hence no need to know the cowl length. The important thing is that your waves (either the Mach waves for the Prandtl-Meyer compression fan case or the shock waves for the 3-oblique-shock case) all meet at the same point and such a point is located exactly at the domain exit.

Question by Student 201327132

Professor, I have a question about design. In problem 3, Should we consider about 3 shocks when I generate grid? I made grid that is consider 3shocks. But I think it is not correct grid. If we should consider about 3 shocks, Would you please some hint for me?

Yes, for problem 3, there should be 3 oblique shocks and the pressure ratio across each shock should be the same. You need to design your inlet so that these 3 shocks appear, have all the same pressure ratio, and meet at one point. I am not sure what you don't understand. Can you explain better the problem?