Computational Aerodynamics Assignment 1 — Euler Equations

The first assignment consists of deriving from basic principles the mass, momentum, and energy transport equations commonly used in CFD.

Question #1

Starting from the principle of conservation of mass, show that the mass conservation equation for a fluid corresponds to:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v_x}{\partial x} + \frac{\partial \rho v_y}{\partial y} + \frac{\partial \rho v_z}{\partial z} = 0$$

with ρ the mass density, and $v_x,\ v_y,\ v_z$ the x, y, z components of the velocity vector.

Question #2

Starting from Newton's law $F_y = m\frac{dv_y}{dt}$ and the mass conservation equation show that the y-component of the momentum transport equation for a fluid corresponds to:

$$\frac{\partial \rho v_y}{\partial t} + \frac{\partial \rho v_x v_y}{\partial x} + \frac{\partial \rho v_y^2}{\partial y} + \frac{\partial \rho v_z v_y}{\partial z} = -\frac{\partial P}{\partial y}$$

with P the pressure.

Question #3

Starting from the 1st law of thermo

$$\mathrm{d}(mh) - V\mathrm{d}P = \delta Q - \delta W$$

the x and y momentum equations in 2D:

$$\rho\frac{\partial v_x}{\partial t} + \rho v_x\frac{\partial v_x}{\partial x} + \rho v_y\frac{\partial v_x}{\partial y} = -\frac{\partial P}{\partial x}$$

$$\rho\frac{\partial v_y}{\partial t} + \rho v_x\frac{\partial v_y}{\partial x} + \rho v_y\frac{\partial v_y}{\partial y} = -\frac{\partial P}{\partial y}$$

show that the total energy transport equation for a fluid corresponds to:

$$\frac{\partial \rho E}{\partial t} + \frac{\partial \rho v_x H}{\partial x} + \frac{\partial \rho v_y H}{\partial y} = 0$$

with the total energy $E \equiv e + \frac{1}{2}q^2$, the total enthalpy $H \equiv h + \frac{1}{2}q^2$, q the speed of the flow, and T the temperature.

Due on Thursday March 21st at 16:30. Do Questions #2 and #3 only.