Computational Aerodynamics Assignment 2 — Generalized Coordinates

Question #1

Starting from the imposed dependencies on the generalized coordinates τ, ξ, and η:

Cartesian Coordinates	Generalized Coordinates
$t = t(\tau)$	$\tau = \tau(t)$
$x = x(\xi, \eta, \tau)$	$\xi = \xi(x, y, t)$
$y = y(\xi, \eta, \tau)$	$\eta = \eta(x, y, t)$

Demonstrate that the metrics of the generalized coordinates correspond to:

$$\xi_t = \frac{\Gamma}{\Omega}(y_\tau x_\eta - x_\tau y_\eta)\,, \quad \xi_x = \frac{y_\eta}{\Omega}, \quad \xi_y = -\frac{x_\eta}{\Omega}$$

and

$$\eta_t = \frac{\Gamma}{\Omega}(x_\tau y_\xi - x_\xi y_\tau)\,, \quad \eta_x = -\frac{y_\xi}{\Omega}, \quad \eta_y = \frac{x_\xi}{\Omega}$$

with $\Gamma \equiv \tau_t$ and Ω the inverse of the metrics Jacobian defined in 2D as:

$$\Omega \equiv x_\xi y_\eta - y_\xi x_\eta$$

Question #2

Starting from the Euler equations

$$\frac{\partial U}{\partial t} + \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} = 0$$

and the metrics η_x, ξ_y, Ω, etc derived above in Question #1, show that the Euler equations can be written in generalized coordinates in strong conservative form as follows:

$$\frac{\partial Q}{\partial \tau} + \frac{\partial G_\xi}{\partial \xi} + \frac{\partial G_\eta}{\partial \eta} = 0$$

with

$$Q \equiv \Omega \Gamma U$$

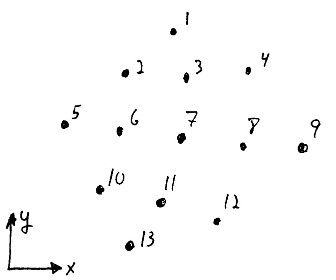
$$G_\xi \equiv \Omega(\xi_x F_x + \xi_y F_y)$$

$$G_\eta \equiv \Omega(\eta_x F_x + \eta_y F_y)$$

Outline clearly your assumptions.

Question #3

Consider the following nodes in the x-y plane:



with the following associated properties:

Node	x, mm	y, mm	ρ, kg/m^3
1	530	-90	1.0
2	400	-210	1.05
3	570	-220	1.05
4	750	-200	1.1
5	220	-360	1.05
6	380	-380	1.1
7	550	-400	1.15
8	730	-410	1.2
9	900	-420	1.25
10	320	-540	1.15
11	500	-580	1.20
12	650	-630	1.25
13	410	-700	1.30

Using the latter, and knowing that

$$F_x = F_y = \rho$$

and with second-order accurate stencils for the metrics and the derivatives do the following:

(a) Find G_η at node 3.

(b) Find G_η at node 11.

(c) Find $\partial G_\eta / \partial \eta$ at node 7.

(d) Find $\partial^2 \rho / \partial x^2$ at node 7.

Question #4

For the nodes shown in Question #3 above, do the following:

(a) Find Ω at node 7 using second-order accurate stencils for the metrics.

(b) Find the cell area at node 7 using a method of your choice and compare it with Ω found in (a).

Due on Thursday March 28th at 16:30. Do Questions #2 and #3 only.