Convective Heat Transfer Assignment 4 — Nusselt and Stanton Number

Question #1

Starting from the energy equation for a constant-ρ and constant-μ fluid:

$$\frac{\partial \rho e}{\partial t}+\frac{\partial \rho u e}{\partial x}+\frac{\partial \rho v e}{\partial y}=\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\mu\left(\frac{\partial u}{\partial x}\right)^2+\mu\left(\frac{\partial u}{\partial y}\right)^2$$
$$+\mu\left(\frac{\partial v}{\partial x}\right)^2+\mu\left(\frac{\partial v}{\partial y}\right)^2$$

Show that the heat transfer for a laminar flow over a constant-temperature flat plate can be expressed as:

$$\mathrm{Nu}_x = 0.332\ \mathrm{Re}_x^{1/2}\ \mathrm{Pr}^{1/3}$$

with Nu_x the Nusselt number, Re_x the Reynolds number and Pr the Prandtl number. Outline all assumptions and also show that the restrictions on the Prandtl number correspond to:

$$0.6 \le \mathrm{Pr} \le 50$$

In doing the derivation, you can use the following boundary layer relationships applicable to laminar flow over a flat plate:

$$\delta\frac{\mathrm{d}\delta}{\mathrm{d}x}=\frac{140}{13}\frac{\mu}{\rho u_\infty}$$

$$\delta^2=\frac{280}{13}\frac{\mu x}{\rho u_\infty}$$

$$\frac{u}{u_\infty}=\frac{3}{2}\frac{y}{\delta}-\frac{1}{2}\left(\frac{y}{\delta}\right)^3$$

Question #2

Starting from the local Nusselt number:

$$\mathrm{Nu}_x = 0.332\,\mathrm{Re}_x^{1/2}\mathrm{Pr}^{1/3}$$

Show that for laminar flow flowing over a flat plate of length L, the average Nusselt number corresponds to:

$$\overline{\mathrm{Nu}_L} = 0.664\,\mathrm{Re}_L^{1/2}\mathrm{Pr}^{1/3}$$

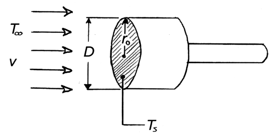
Question #3

Experiments to determine the local convection heat transfer coefficient for uniform flow normal to a heated circular disk have yielded a radial Nusselt number distribution of the form

$$\mathrm{Nu}_D=\frac{h(r)D}{k}=\mathrm{Nu}_0\left[1+a\left(\frac{r}{r_0}\right)^n\right]$$

where both n and a are positive. The Nusselt number at the stagnation point is correlated in terms of the Reynolds number ($\mathrm{Re}_D=\rho v D/\mu$) and Prandtl number ($\mathrm{Pr}=c_p\mu/k$):

$$\mathrm{Nu}_0=\frac{h(r=0)D}{k}=0.814\,\mathrm{Re}_D^{0.5}\,\mathrm{Pr}^{0.36}$$



Obtain an expression for the average Nusselt number, $\overline{\mathrm{Nu}_D}=\bar{h}D/k$, corresponding to heat transfer from an isothermal disk.

Answers

1.
2.
3. $\overline{\mathrm{Nu}_D} = 2\mathrm{Nu}_0\,(0.5 + a/(n+2))$.

Due on Monday April 23rd at 5:00pm. Do all questions.