Convective Heat Transfer Assignment 2 — Viscous Dissipation

Question #1

Derive Fourier's law of heat conduction in a gas:

$$q_x'' = -k\frac{\partial T}{\partial x}$$

with

$$k = \frac{5k_{\rm B}}{4\sigma}\sqrt{\frac{3RT}{2}}$$

with k the thermal conductivity, σ the collision cross-section, $k_{\rm B}$ the Boltzmann constant and R the gas constant.

Question #2

Starting from the 1st law of thermo

$${\rm d}(mh) - V{\rm d}P = \delta Q - \delta W$$

the y momentum equation in 1D

$$\rho\frac{\partial v}{\partial t} + \rho v\frac{\partial v}{\partial y} = -\frac{\partial P}{\partial y} + \frac{\partial \tau_{yy}}{\partial y}$$

and Fourier's law $\vec{q}'' = -k\vec{\nabla}T$, show that the total energy transport equation for a viscous fluid corresponds to:

$$\frac{\partial \rho E}{\partial t} + \frac{\partial \rho v H}{\partial y} = \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial v\tau_{yy}}{\partial y}$$

with the total energy $E \equiv e + \frac{1}{2}q^2$, the total enthalpy $H \equiv h + \frac{1}{2}q^2$, q the speed of the flow, k the thermal conductivity, and T the temperature.

Question #3

Starting from the energy equation

$$\rho\frac{\partial E}{\partial t} + \rho u\frac{\partial H}{\partial x} + \rho v\frac{\partial H}{\partial y} = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial u\tau_{xx}}{\partial x} + \frac{\partial u\tau_{yx}}{\partial y} + \frac{\partial v\tau_{xy}}{\partial x} + \frac{\partial v\tau_{yy}}{\partial y}$$

the x momentum equation

$$\rho\frac{\partial u}{\partial t} + \rho u\frac{\partial u}{\partial x} + \rho v\frac{\partial u}{\partial y} = -\frac{\partial P}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y}$$

and the y momentum equation

$$\rho\frac{\partial v}{\partial t} + \rho u\frac{\partial v}{\partial x} + \rho v\frac{\partial v}{\partial y} = -\frac{\partial P}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y}$$

Show that the energy equation for a constant-ρ and constant-μ fluid corresponds to:

$$\rho\frac{\partial e}{\partial t} + \rho u\frac{\partial e}{\partial x} + \rho v\frac{\partial e}{\partial y} = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \phi$$

with ϕ the viscous dissipation per unit volume defined as:

$$\phi \equiv \mu\left(\frac{\partial u}{\partial x}\right)^2 + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \mu\left(\frac{\partial v}{\partial x}\right)^2 + \mu\left(\frac{\partial v}{\partial y}\right)^2$$

Question #4

Consider two large (infinite) parallel plates, 5 mm apart. One plate is stationary, while the other plate is moving at a speed of 200 m/s. Both plates are maintained at 27° C. Consider two cases, one for which the plates are separated by water and the other for which the plates are separated by air.

(a) For each of the two fluids, what is the force per unit surface area required to maintain the above condition? What is the corresponding power requirement?

(b) What is the viscous dissipation associated with each of the two fluids?

(c) What is the maximum temperature in each of the two fluids?

Answers

1.
2.
3.
4. 0.74 N/m^2, 34.4 N/m^2, 148 W/m^2, 6880 W/m^2, 29.6 kW/m^3, 1.376 MW/m^3, 30.5° C, 34.0° C.

Due on Wednesday April 4th at 17:00. Do Questions 1, 3, and 4 only.