Fundamentals of Fluid Mechanics A Assignment 1 — Clapeyron, Pascal, and Archimedes

Instructions

Write your solutions in single column format, with one statement following another vertically. Write your solutions neatly so that they are easy to read and verify. Don't write one line with two equal signs. Highlight your answers using a box. Failure to do this will result in a lower score and fewer comments on my part.

Question #1

Starting from $\vec{F} = m\vec{a}$ applied on a gas particule, show Clapeyron's ideal gas law:

$$P = \rho RT = Nk_{\mathrm{B}}T = \frac{RT}{v}$$

with

$$T \equiv \frac{m\overline{q^2}}{3k_{\mathrm{B}}}, \quad R \equiv \frac{k_{\mathrm{B}}}{m}$$

Outline the definition of the pressure P, the density ρ, the specific volume v, and the number density N.

Question #2

Starting from the balance of the forces on a fluid at rest prove Pascal's law $\Delta P = -\rho g \Delta y$ with g the gravitational acceleration constant, Δy the change in height, and ΔP the change in pressure. List all assumptions and define clearly all terms used.

Question #3

Starting from the balance of the forces on a fluid at rest prove Archimedes's principle $\vec{F}_{\mathrm{b}} = -\rho\vec{g}V$ with V the volume of the displaced fluid, $\vec{g}$ the gravitational acceleration vector and ρ the density of the displaced fluid. List all assumptions and define clearly all terms used.

Question #4

Show through integration that the force that a constant pressure P exerts on a hemisphere of radius R is of $P\pi R^2$.

Question #5

Show that the average of a sum of terms is the sum of the averages:

$\overline{\overline{a} + \overline{b}} = \overline{a} + \overline{b}$

Due on September 9th at 11:00. Do Questions #2, #3, and #4 only.